Answer on Question #65498, Physics / Mechanics | Relativity

Question:

A proton undergoes a head on elastic collision with a particle of unknown mass which is initially at rest and rebounds with 16/25 of its initial kinetic energy. Calculate the ratio of the unknown mass with respect to the mass of the proton.

Solution:

Kinetic energy of the proton before collision $E_1^k = \frac{m_p v_1^2}{2}$, where m_p is proton's mass.

Kinetic energy of the proton after collision $E_2^k = \frac{m_p v_2^2}{2} = \frac{16}{25} E_1^k$.

So $\frac{m_p v_2^2}{2} = \frac{16}{25} \cdot \frac{m_p v_1^2}{2}$, and we may evaluate $v_2 = \frac{4}{5} v_1$.

Assuming that the system is closed we may use the momentum conservation law:

$m_p \vec{v}_1 + \vec{0} = m_p \vec{v}_2 + m_x \vec{v}_x$, where m_x is unknown particle's mass.

Or in scalar form, $m_p v_1 = -m_p v_2 + m_x v_x$.

$$m_p v_1 = -m_p \frac{4}{5} v_1 + m_x v_x \Rightarrow \frac{9}{5} m_p v_1 = m_x v_x \Rightarrow v_x = \frac{9}{5} \frac{m_p}{m_x} v_1$$

Next, according to the law of conservation of energy, $E_1^k = E_2^k + E_x^k$.

$$\frac{m_p v_1^2}{2} = \frac{m_p v_2^2}{2} + \frac{m_x v_x^2}{2} \Rightarrow \frac{m_p v_1^2}{2} = \frac{m_p \left(\frac{4}{5} v_1\right)^2}{2} + \frac{m_x \left(\frac{9}{5} \frac{m_p}{m_x} v_1\right)^2}{2} \Rightarrow$$

$$m_p v_1^2 = \frac{16}{25} m_p v_1^2 + \frac{81}{25} m_x \left(\frac{m_p}{m_x}\right)^2 v_1^2 \Rightarrow \frac{9}{25} m_p = \frac{81}{25} m_x \left(\frac{m_p}{m_x}\right)^2 \Rightarrow$$

$$\frac{m_p}{m_x} = 9 \left(\frac{m_p}{m_x}\right)^2 \Rightarrow \frac{m_p}{m_x} = \frac{1}{9} \Rightarrow \frac{m_x}{m_p} = 9$$

Answer:

$$\frac{m_x}{m_p} = 9$$

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