

Answer on Question 65307, Physics, Electric Circuits

Question:

Two capacitors have an equivalent capacitance of 16.0 pF when they are hooked up in parallel. The same two capacitors when hooked up in series have an equivalent capacitance of 1.75 pF . What is the capacitance of each capacitor?

Solution:

Let the capacitance of the first capacitor be C_1 and the second one - C_2 . Then, we can write the equivalent capacitance of two capacitors connected in series or in parallel:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}, (1)$$

$$C_p = C_1 + C_2, (2)$$

here, $C_s = 1.75 \text{ pF}$ is the equivalent capacitance of two capacitors connected in series, $C_p = 16.0 \text{ pF}$ is the equivalent capacitance of two capacitors connected in parallel.

Let's express C_2 from the equation (2) and substitute it into the equation (1):

$$C_2 = C_p - C_1,$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1},$$

$$\frac{1}{C_s} = \frac{C_p}{C_1(C_p - C_1)}.$$

After simplification we get the quadratic equation:

$$C_1^2 - C_1 C_p + C_p C_s = 0.$$

Let's substitute C_s and C_p into the equation:

$$C_1^2 - 16C_1 + 28 = 0.$$

This quadratic equation has two roots:

$$C_1 = \frac{16 + \sqrt{(16)^2 - 4 \cdot 28}}{2} = 14,$$

$$C_1 = \frac{16 - \sqrt{(16)^2 - 4 \cdot 28}}{2} = 2.$$

Both roots of the quadratic equation have the sense. If we have $C_1 = 14 \text{ pF}$, then we should take $C_2 = 2 \text{ pF}$ to obtain $C_s = 1.75 \text{ pF}$ and $C_p = 16.0 \text{ pF}$. Similarly, if we have $C_1 = 2 \text{ pF}$, then we should take $C_2 = 14 \text{ pF}$ and we obtain the same C_s and C_p . Therefore, the capacitance of each capacitor is 14 pF and 2 pF .

Answer:

The capacitance of each capacitor is 14 pF and 2 pF .