Answer on Question #65252-Physics-Other

Two balls of radius r and their centers separated by distance d are given charge +q and -q respectively. Find capacitance of this two ball system.

Solution

Let s be the distance from the center of the first ball. The electric field is

$$E(s) = \frac{q}{4\pi\epsilon_0 s^2} - \frac{-q}{4\pi\epsilon_0 (d-s)^2}$$

The potential difference is

$$U = \int_r^{d-r} E(s)ds = \frac{q1}{4\pi\epsilon_0}\int_r^{d-r}\left[\frac{1}{s^2} + \frac{1}{(s-d)^2}\right]ds = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{d-s} - \frac{1}{s}\right]_r^{d-r}$$
$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{d-d+r} - \frac{1}{d-r} - \frac{1}{d-r} + \frac{1}{r}\right] = \frac{q}{2\pi\epsilon_0}\frac{d-2r}{r(d-r)}$$

The capacitance of this two ball system is

$$C = \frac{q}{U} = 2\pi\epsilon_0 r\left(\frac{d-r}{d-2r}\right)$$