

A football quarterback throws a pass at an angle of 32.5° . He releases the pass 3.50 m behind the line of scrimmage. His receiver left the line of scrimmage 2.1 s earlier, going straight down-field at a constant speed of 7.50 m/s. With what speed must the quarterback throw the ball so that the pass lands gently in the receiver's hands without the receiver breaking stride? Assume that the ball is released at the same height it is caught and that the receiver is straight downfield from the quarterback at the time of release. (Ignore any effects due to air resistance.)

Motion of ball is described by the system

$$\begin{cases} x_b = V_b \cos \alpha t \\ y_b = V_b \sin \alpha t - \frac{gt^2}{2} \end{cases}$$

When the ball fall at the ground $y=0$. So

$$y = V_b \sin \alpha t - \frac{gt^2}{2} = 0 \rightarrow \begin{cases} t = 0 \\ t = \frac{2V_b \sin \alpha}{g} \end{cases}$$

Motion of receiver is described by equation

$$x_r = V_r (t + 2.1)$$

When receiver catch the ball

$$x_b = x_r \rightarrow V_b \cos \alpha t = V_r (t + 2.1) \rightarrow t = \frac{2.1V_r}{V_b \cos \alpha - V_r}$$

Then we have

$$\frac{2V_b \sin \alpha}{g} = \frac{2.1V_r}{V_b \cos \alpha - V_r} \rightarrow 2V_b^2 \sin \alpha \cos \alpha - 2V_b V_r \sin \alpha - 2.1V_r g = 0 \rightarrow$$

$$V_b = \frac{2V_r \sin \alpha + \sqrt{4V_r^2 \sin^2 \alpha + 16.2V_r g \sin \alpha \cos \alpha}}{4 \sin \alpha \cos \alpha} = 15.38 \frac{m}{s}$$