Answer on Question 64335, Physics, Mechanics, Relativity

Question:

A 60 kg woman stands at the rim of a horizontal turntable having a moment of inertia of $500\ kg \cdot m^2$ and a radius of $2\ m$. The turntable is initially at rest and is free to rotate about a frictionless, vertical axle through its center. The woman then starts walking around the rim clockwise (as viewed from above the system) at a constant speed of 1.5 m/s relative to the Earth.

(a) In what direction and with what angular speed does the turntable rotates?

(b) How much work does the woman do to set herself and the turntable in motion?

Solution:

(a) We can find the angular speed of the turntable from the law of conservation of angular momentum. It states that the total angular momentum of a system is constant in both magnitude and direction if the net external torque acting on the system is zero. Since initially the system of turntable and woman is at rest and there is no external torque on the turntable we can write:

$$L_i = L_f,$$

$$L_{woman} + L_{turntable} = 0,$$

$$I_{woman}\omega_{woman} + I_{turntable}\omega_{turntable} = 0,$$

here, L_i is the initial angular momentum of the system, L_f is the final angular momentum of the system, L_{woman} is the angular momentum of the woman, $L_{turntable}$ is the angular momentum of the turntable, I_{woman} is the moment of inertia of the woman, $I_{turntable}$ is the moment of inertia of the turntable, ω_{woman} is the angular speed of the woman and $\omega_{turntable}$ is the angular speed of the turntable.

Then, we get:

$$I_{turntable}\omega_{turntable} = -I_{woman}\omega_{woman},$$

$$\omega_{turntable} = -\frac{I_{woman}}{I_{turntable}} \cdot \omega_{woman.}$$

We can find the moment of inertia of the woman from the formula:

$$I_{woman} = m_{woman}r^2,$$

here, m_{woman} is the mass of the woman, r is the radius of the turntable.

In order to find the angular speed of the woman we need to use the relationship between the linear and angular quantities:

$$v_{woman} = r\omega_{woman},$$

$$\omega_{woman} = \frac{v_{woman}}{r},$$

here, v_{woman} is the linear speed of the woman.

Finally, substituting I_{woman} and ω_{woman} into the formula for the angular speed of the turntable, we get:

$$\omega_{turntable} = -\frac{I_{woman}}{I_{turntable}} \cdot \omega_{woman} = -\frac{m_{woman}r^2 \cdot \left(\frac{v_{woman}}{r}\right)}{I_{turntable}} =$$

$$= -\frac{m_{woman} r v_{woman}}{I_{turntable}} = -\frac{60\ kg \cdot 2\ m \cdot 1.5\ \frac{m}{s}}{500\ kg \cdot m^2} = -0.36\ \frac{rad}{s}.$$

The sign minus indicates that the turntable rotates in the counter-clockwise direction.

(b) We can find how much work does the woman do to set herself and the turntable in motion from the work-kinetic energy theorem:

$$W = \Delta KE_{tot} = KE_f - KE_i.$$

Since $KE_i = 0$ (initially the system of woman and turntable is at rest), we can write:

$$W = KE_f = KE_{translational} + KE_{rotational},$$

$$W = \frac{1}{2} m_{woman} v_{woman}^2 + \frac{1}{2} I_{turntable} \omega_{turntable}^2 =$$

$$= \frac{1}{2} \cdot 60\ kg \cdot \left(1.5\ \frac{m}{s}\right)^2 + \frac{1}{2} \cdot 500\ kg \cdot m^2 \cdot \left(0.36\ \frac{rad}{s}\right)^2 = 99.9\ J.$$

Answer:

(a) $\omega_{turntable} = 0.36\ \frac{rad}{s}$.

(b) $W = 99.9\ J$.