Answer on Question #64243-Physics-Quantum Mechanics

Calculate the probability that an electron in the ground state of the Hydrogen atom is outside the classically allowed region, i.e. (r > 2a0).

Solution

The probability of finding an electron in the forbidden region is:

$$P = \int\limits_{forbidden\ region} d^3\boldsymbol{r}\ |\psi_{100}(\boldsymbol{r})|^2 = \int\limits_{2a_0}^{\infty} dr r^2 \int d\Omega\ |\psi_{100}(\boldsymbol{r})|^2$$

$$= \int\limits_{2a_0}^{\infty} dr r^2 \int d\Omega \left[\frac{1}{\sqrt{4\pi}}(2)\left(\frac{1}{a_0}\right)^{\frac{3}{2}} e^{-\frac{r}{a_0}} \right]^2 = \frac{4}{a_0^3} \int\limits_{2a_0}^{\infty} dr r^2 e^{-\frac{2r}{a_0}}$$

$$= \frac{4}{a_0^3} \left[e^{-\frac{2r}{a_0}} \left(-\frac{a_0 r^2}{2} - \frac{a_0^2 r}{2} - \frac{a_0^3}{4} \right) \right]_{2r_0}^{\infty} = \frac{4}{a_0^3} e^{-4} \left(2a_0^3 + a_0^3 + \frac{a_0^3}{4} \right) = 13e^{-4} \approx 0.24.$$

Answer: $13e^{-4} \approx 0.24$.

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