Answer on Question #64137, Physics / Mechanics | Relativity

1. Two posts, one 8ft high and the other 12ft high, stand 15ft apart. They are to be stayed by wires attached to a single stake at ground level, the wires running to the tops of the posts. Where the stake should be placed, to use the least amount of wire?

Solution:

Therefore, using Pythagoras', the lengths of the wires are

$\sqrt{(x^2 + 8^2)}$ and $\sqrt{((15-x)^2 + 12^2)}$

Therefore the total length

$L = \sqrt{(x^2 + 64)} + \sqrt{(369 - 30x + x^2)}$

Differentiate using 'function of a function' (chain rule):

$L' = x/\sqrt{(x^2 + 64)} + (x - 15) / \sqrt{(369 - 30x + x^2)}$

.. = 0 when

$x/\sqrt{(x^2 + 64)} = -(x - 15) / \sqrt{(369 - 30x + x^2)}$

Hence

$x \sqrt{(369 - 30x + x^2)} = (15 - x)\sqrt{(x^2 + 64)}$

Square both sides:

$x^2(369 - 30x + x^2) = (15 - x)^2(x^2 + 64)$

$369x^2 - 30x^3 + x^4 = (225 - 30x + x^2)(x^2 + 64)$

$369x^2 - 30x^3 + x^4 = x^4 - 30x^3 + 289x^2 - 1920x + 14400$

$80x^2 + 1920x - 14400 = 0$

$x^2 + 24x - 180 = 0$

$(x + 30)(x - 6) = 0$

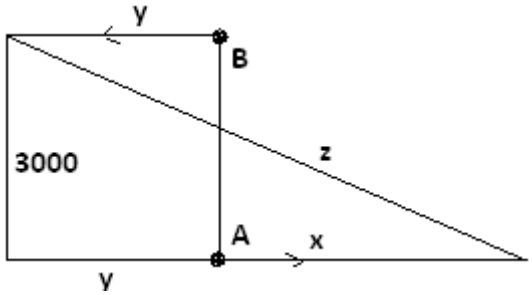
Since $x > 0$,

$x = 6$

Answer: 6

2. A man starts walking eastward at 5ft/sec from a point A. Ten minutes later a second man starts walking west at the rate of 5ft/sec from a point B, 3000ft north of A. How fast are they separating 10 minutes after the second man starts?

Solution:



$[dz/dt]_{t= 1200}$

$dx/dt = 5$

$dy/dt = 5$

$z = \sqrt{(3000)^2 + (x+y)^2}$

$dz/dt = ½ [3000^2 + (x+y)^2]^{-1/2} 2 (x+y) (dx/dt + dy/dt)$

$t = 1200$

$x = 5 (1200) = 6000$

$y = 5 (600) = 3000$

$[dz/dt]_{t= 1200} = [3000^2 + (6000 + 3000)^2]^{-1/2} 2 (6000 + 300) (5 + 5) = \sqrt{90}$ ft/sec

3. A man 6 feet tall walks away from the point under the light at the rate of 5ft/sec. How fast is his shadow lengthening when he is 20 feet away from the point under the light?

Solution:

dx/dt = 5

20/6 = y /(y-x)

20y – 20x = 6y

14y = 20x

y = 20x /14

y = 10x / 7

dy/dt = (10/7) dx/dt

dy/dt = (10/7) 5 = 50 / 5 ft/sec

d(y – X)/dt = dy/dt - dx/dt = (50 / 5) – 5 = 15/7 ft/sec

Answer: 15/7 ft/sec

4. At a given instant the legs of a right triangle are 8 in., and 6 in., respectively. The first leg decreases at 1 in/min and the second increases at 2 in/min. At what rate is the area increasing after 2 min.?

Solution:

A= 1/2bh

db /d t =-1

dh/d t =2

The attempt at a solution

A= 1/2bh

dA /d t = 1/2 (db/dt*h+b* dh/dt)

dA/dt= 1/2(-1*10+6*2)

dA/dt= 1/2 (2)= 1

Answer: 1