A satellite's orbit velocity 1 meter from the surface of a smooth spherical planet is 23640 km/hr. It is known that the planet's average density is 15.2 gm/(cm^3). Determine the radius of the planet in km.

Solution

The orbital velocity formula is

$$v = \sqrt{\frac{GM}{r+h}}.$$

$$M = \frac{4}{3}\pi r^3 \rho.$$

$$v^2 = \frac{G\frac{4}{3}\pi r^3 \rho}{r+h} \rightarrow G\frac{4}{3}\pi r^3 \rho - v^2 r - v^2 h = 0$$

$$\rho = 15\,200\,\frac{kg}{m^3}.$$

$$G = 6.67 \cdot 10^{-11}\frac{Nm^2}{kg^2}.$$

$$v = \frac{23640}{3.6}\frac{m}{s}$$

The final equation is

$$6.67 \cdot 10^{-11} \cdot \frac{4}{3}\pi \cdot 15\,200 \cdot r^3 - \left(\frac{23640}{3.6}\right)^2 r - \left(\frac{23640}{3.6}\right)^2 \cdot 1 = 0$$

We have 3 answers:

$$r_1 = -1$$

$$r_{2,3} = \pm 7.17 \cdot 10^6\,m.$$

The radius cannot be negative, so

$$r = 7.17 \cdot 10^6\,m = 7170\,km.$$

Answer: 7170 km.