Answer on Question #63140, Physics / Other

When a sound wave give a certain intensity is detected by a the tympanic membrane (eardrum) the amplitude of the resultant motion is 1.0nm (1.0x10^-9m). If the frequency of the aound is 600 Hz, what is the maximum speed of the membrane oscillation?

SOLUTION

Simple harmonic motion is a type of periodic motion where the restoring force is directly proportional to the displacement and acts in the direction opposite to that of displacement. It can be described by the function

$$x(t) = A\cos(\omega t + \varphi),$$

where x is an oscillation variable (displacement from the equilibrium position), *A* is the amplitude (maximum displacement from the equilibrium position), $\omega = 2\pi f$ is the angular frequency, and φ is the phase. The velocity as a function of time can be found as a derivative of x:

$$v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi).$$

In our particular situation *x* is a displacement of eardrum from its equilibrium position (look at figure 1). Therefore,

A=1.0 nm,

$\omega = 2\pi f = 2\pi \cdot 600\ rad/s = 1200\pi\ rad/s.$

We are no interested in phase, let it be zero ($\varphi = 0$).

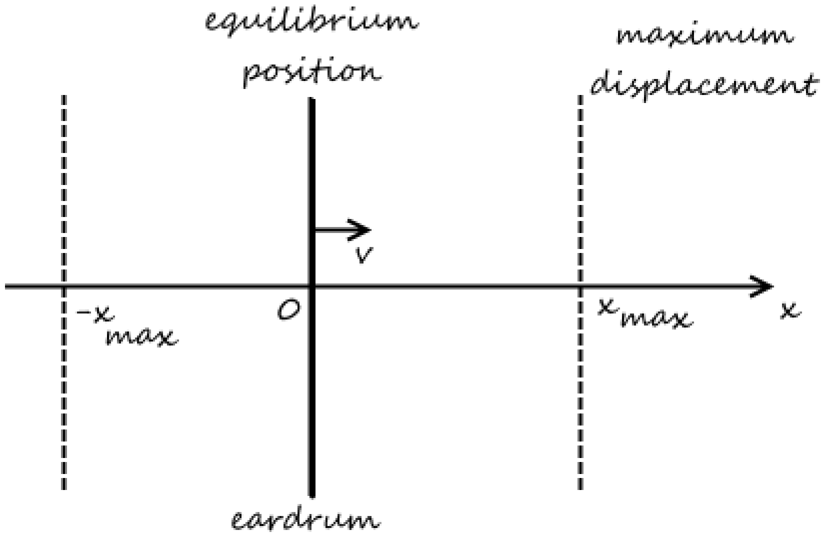

Figure 1. $x_{max}=A$

So, maximum speed will be $v_m = A\omega$ because of limiting sine of 1. It happens at equilibrium point. That's why

$$v_m = A \cdot 2\pi f = 2\pi \cdot 1nm \cdot 600\,Hz = 1200\,s^{-1} \cdot \pi \cdot 1.0 \cdot 10^{-9} m = 1.2 \cdot \pi \cdot 10^{-6}\,m/s \approx 3.77\ \mu m/s.$$

Figure 2 shows a behavior of x and v.

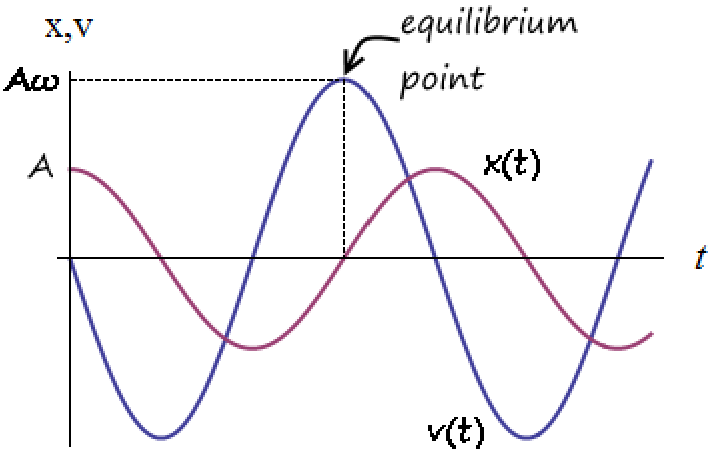

Figure 2.

ANSWER: the maximum speed of the membrane oscillation is $3.77\ \mu m/s$ at the equilibrium point.