Answer on Question 62782, Physics, Mechanics, Relativity

Question:

In the vertical jump, an Kobe Bryant starts from a crouch and jumps upward to reach as high as possible. Even the best athletes spend little more than $1.00\ s$ in the air (their "hang time"). Treat Kobe as a particle and let y_{max} be his maximum height above the floor. To explain why he seems to hang in the air, calculate the ratio of the time he pends above $y_{max}/2$ to the time it takes him to go from the floor to that height. You may ignore air resistance.

Solution:

Let's first find the velocity v_1 of an athlete to reach half of the maximum height from the kinematic equation:

$$v_1^2 = v_0^2 - 2gh = v_0^2 - 2g\left(\frac{y_{max}}{2}\right),$$

here, v_0 is the initial velocity of an athlete, v_1 is the velocity of an athlete at half the maximum height, g is the acceleration due to gravity, $h = y_{max}/2$ is the half of the maximum height.

We can find the maximum height that an athlete can reach from the Law of Conservation of Energy:

$$KE = PE,$$

$$\frac{1}{2}mv_0^2 = mgy_{max},$$

$$y_{max} = \frac{v_0^2}{2g}.$$

Then, substituting y_{max} into the first equation we get:

$$v_1^2 = v_0^2 - 2g\left(\frac{v_0^2}{4g}\right),$$

$$v_1^2 = v_0^2 - \frac{v_0^2}{2} = \frac{v_0^2}{2},$$

$$v_1 = \frac{v_0}{\sqrt{2}}.$$

We can find the time t that an athlete needs to reach the maximum height (y_{max}) from the kinematic equation:

$$v = v_0 - gt,$$

here, v is the final velocity of an athlete at the maximum height, v_0 is the initial velocity of an athlete.

Since, $v = 0\ ms^{-1}$, we get:

$$t = \frac{v_0}{g}.$$

Similarly, we can find the time t_1 that an athlete needs to reach maximum height from the ($y_{max}/2$):

$$t_1 = \frac{v_1}{g} = \frac{v_0}{g\sqrt{2}}$$

So, it is obviously, that the time to reach y_{max} from $y_{max}/2$ (or the time he is above $y_{max}/2$ moving up) is nothing more than the difference between t and t_1:

$$t - t_1 = \frac{v_0}{g}\left(1 - \frac{1}{\sqrt{2}}\right),$$

Finally, we can calculate the ratio of the time he is above $y_{max}/2$ to the time it takes him to go from the floor to that height:

$$\frac{t_1}{t - t_1} = \frac{v_0}{g\sqrt{2}} \frac{g\sqrt{2}}{v_0\left(\sqrt{2} - 1\right)} = 2.4$$

Answer:

The athlete spends 2.4 times more time at the upper part of his way than in the lower one.