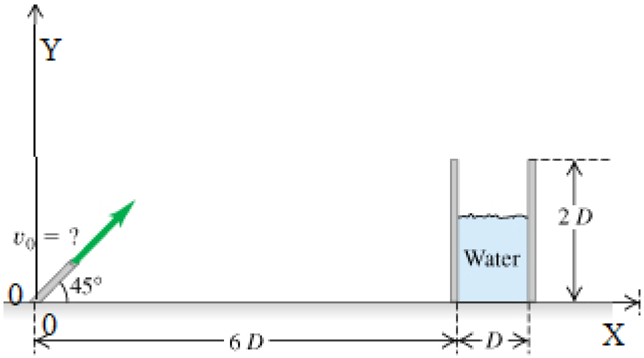
Answer on Question #62487-Physics-Other

A water hose is used to fill a large cylindrical storage tank of diameter D and height 20. The hose shoots the water at 45° above the horizontal from the same level as the base of the tank and is a distance 6 D away. For what range of launch speeds (vo) will the water enter to the tank? Ignore air resistance, and express your answer in terms of D and g

Solution



Let the nozzle of the hose be at the origin. Then the nearest part of the rim of the tank is at $(x, y) = (6D, 2D)$ and the furthest part of the rim is at $(x, y) = (7D, 2D)$.

The trajectory of the water can be found as follows:

$x(t) = \frac{\sqrt{2}}{2} v_0 t$ (cos(45) $=\frac{\sqrt{2}}{2}$)

$y(t) = \frac{\sqrt{2}}{2} v^0 t - \frac{1}{2} g t^2$. (sin(45) $=\frac{\sqrt{2}}{2}$)

Substituting the first ($t = \frac{\sqrt{2}x}{v_0}$) into the second gives the trajectory:

$$y = x - \left(\frac{g}{v_0^2}\right) x^2$$

Now we want y to be equal to 2D when x is between 6D and 7D, but also that we are on the downward path of the parabola there.

Now

$x - \left(\frac{g}{v_0^2}\right) x^2 = 2\, D$ can be rewritten as

$$\left(\frac{g}{v_0^2}\right) x^2 - x + 2\, D = 0$$

with solutions:

$$x = \frac{v_0^2}{2g} \pm \frac{v_0^2}{2g} \sqrt{\left(1 - \frac{8gD}{v_0^2}\right)}$$

But only the largest solution corresponds to the downward part of the parabola.

So we know that

$$6D < \frac{v_0^2}{2g} + \frac{v_0^2}{2g}\sqrt{\left(1 - \frac{8gD}{v_0^2}\right)} < 7D$$

Solving

$$12gD - v_0^2 < v_0^2\sqrt{\left(1 - \frac{8gD}{v_0^2}\right)} < 14gD - v_0^2$$

$$(12gD - v_0^2)^2 < v_0^4 - 8gD\sqrt{\left(1 - \frac{8gD}{v_0^2}\right)} < 14gD - v_0^2)^2$$

Left inequality:

$$v_0{}^4 - 24\, g\, D\, v_0^2 + (12\, gD)^2 < v_0{}^4 - 8gD\ v_0^2$$

$$-16\, g\, D\ v_0^2 < -144\, g^2 D^2$$

$$v_0 > \sqrt{(9\, g\, D)}$$

Right inequality:

$$v_0^4 - 8gDv_0^2 < v_0^4 + (14gD)^2 - 28\, g\, D\, v_0^2$$

$$20\, g\, D\, v_0^2 < 196\, g^2 D^2$$

$$v_0 < \sqrt{\frac{49}{5} gD}$$

So the answer is:

$$\sqrt{9\, g\, D} < v_0 < \sqrt{\frac{49}{5} gD}$$