Answer on Question 62453, Physics, Atomic and Nuclear Physics

Question:

Calculate the mass defect and binding energy per nucleon for a lithium nucleus ($^{7}_{3}Li$):

Mass of the lithium nucleus $M = 7.0\ u$

Mass of the proton $m_p = 1.007825\ u$

Mass of the neutron $m_n = 1.008665\ u$

$1\ u = 1.6605 \cdot 10^{-27} kg = 931\ MeV$.

Solution:

a) The mass of the nucleus of an atom of any element is always found to be less then the sum of the masses of its constituent nucleons. This difference in mass is called the mass defect. Mathematically it can be written as follows:

$$\Delta m = \left(Z \cdot m_p + N \cdot m_n\right) - M,$$

here, Z is the number of protons, N is the number of neutrons, m_p is the mass of the proton, m_n is the mass of the neutron and M is the mass of the nucleus of an atom.

Let's substitute the numbers and find Δm:

$$\Delta m = \left(Z \cdot m_p + N \cdot m_n\right) - M = (3 \cdot 1.007825\ u + 4 \cdot 1.008665\ u) - 7.0\ u =$$
$$= 3.023475\ u + 4.03466\ u - 7.0\ u = 0.058135\ u =$$
$$= 0.058135 \cdot 1.6605 \cdot 10^{-27} kg = 9.65 \cdot 10^{-29}\ kg.$$

b) Let's first use the famous Einstein formula to find the binding energy of the nucleons:

$$E = \Delta mc^2 = \left[\left(Z \cdot m_p + N \cdot m_n\right) - M\right] \cdot c^2 =$$
$$= 0.058135 \cdot 1.6605 \cdot 10^{-27} kg \cdot \left(3.0 \cdot 10^8\ \frac{m}{s}\right)^2 = 8.685 \cdot 10^{-12}\ J =$$
$$= 54.28\ MeV.$$

$^{7}_{3}Li$ has 7 nucleons (3 protons and 4 neutrons). Then, the binding energy per nucleon will be:

$$E_{per\ nucleon} = \frac{E}{A} = \frac{E}{Z + N} = \frac{54.28\ MeV}{7} = 7.754\ MeV.$$

Answer:

a) $\Delta m = 9.65 \cdot 10^{-29}\ kg$.

b) $E_{per\ nucleon} = 7.754\ MeV$.