Answer on Question #62021, Physics / Optics

Show that the superposition of two linearly polarized waves having different amplitude and a finite phase difference can used to be produced for an elliptically polarized light waves.

Solution:

The first wave: $E_x = E_{0x}\sin(\omega t + \delta)$ (1)

The second wave: $E_y = E_{0y}\sin(\omega t)$ (2)

If δ be the phase difference between the two emergent beams from second equation we have:

$$\frac{E_y}{E_{oy}} = \sin(\omega t)$$

Hence $\cos\omega t = \sqrt{1-\sin^2\omega t} = \sqrt{1-\left(\frac{E_y}{E_{oy}}\right)^2}$

From first eqation we have:

$$E_x = E_{0x}\sin(\omega t + \delta\ ) = E_{0x}(\sin\omega t\cos\delta\ + \cos\omega t\sin\delta\ )$$

or,

$$\frac{E_x}{E_{0x}} = \sin\omega t\cos\delta\ + \cos\omega t\sin\delta\ = \frac{E_y}{E_{oy}}\cos\delta + \sqrt{1-\left(\frac{E_y}{E_{oy}}\right)^2}\sin\delta$$

or,

$$\frac{E_x}{E_{0x}} - \frac{E_y}{E_{oy}}\cos\delta = \sqrt{1-\left(\frac{E_y}{E_{oy}}\right)^2}\sin\delta$$

Squaring and rearranging, we get:

$$\left(\frac{E_x}{E_{ox}}\right)^2 + \left(\frac{E_y}{E_{oy}}\right)^2 - \frac{2E_xE_y}{E_{0x}E_{0y}}\cos\delta = \sin^2\delta$$

This is the equation of an ellipse.

This is the equation for an ellipse. What it shows is that at any instant of time the locus of points described by the propagation of E_x and E_y will trace out this curve.