Answer on Question #62015, Physics / Optics

An object is placed 100 cm in front of a thin lens with focal length 40 cm, which is 200 cm in front of plane mirror. Where is the final image measured from the lens, that would be seen by an eye looking toward the mirror through the lens. What is the transverse magnification of the image? (The light goes from the object through the lens, reflected of the mirror and goes through the lens again.)

Solution:

First, find the location of the image created by the lens by itself (as if no other instruments were present).

The thin lens equation:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

where o = object distance, i = image distance, f = focal length.

Hence,

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{0.4\,m}-\frac{1}{1\,m}=2.5-1=1.5$$

$$d_i=\frac{1}{1.5}=0.667\ \text{m}$$

Next, find the location of the image created by the plane mirror (after the light passes through the lens). The object distance from the mirror is

$$d_2=2.0\ \text{m}-0.667\ \text{m}=1.333\ \text{m}$$

Then, since for a plane mirror the image distance is equal to the object distance from the mirror, we have that the image measured from the lens is placed at

$$d_3=2.0+1.333=3.333\ \text{m}$$

The light reflects off of the mirror and back through the lens a second time!

Thus,

$$\frac{1}{d_{i2}}=\frac{1}{f}-\frac{1}{d_3}=\frac{1}{0.4}-\frac{1}{3.333}=2.2$$

Thus the final image measured from the lens placed at

$$d_{i2}=0.4545\ \text{m}$$

The linear or transverse magnification of a thin lens is

$$M=\frac{f}{f-d_o}$$

where f is the focal length and d_o is the distance from the lens to the object.

First, find the magnitude M1 of the magnification of the image created when light from the object passes through the lens the first time (as if the mirror were not present).

$$M_1 = \frac{0.4}{0.4 - 1} = \frac{0.4}{-0.6} = -0.667$$

Notice the sign convention of the magnification shows that, if m is negative, as it is for real images, the image is upside-down with respect to the object.

Plane mirrors don't magnify your image!

Now find the magnitude M2 of the magnification of the image created when light from the object passes through the lens the second time (after reflecting off of the mirror).

$$M_2 = \frac{0.4}{0.4 - 3.333} = -0.1364$$

If you have more than one optical instrument, the total magnification is equal to the product of the individual magnifications.

$$M = M_1 \cdot M_2 = 0.667 \cdot 0.1364 = 0.09$$

Answer: $d_{final} \approx 45.5$ cm; $M = 0.09$.