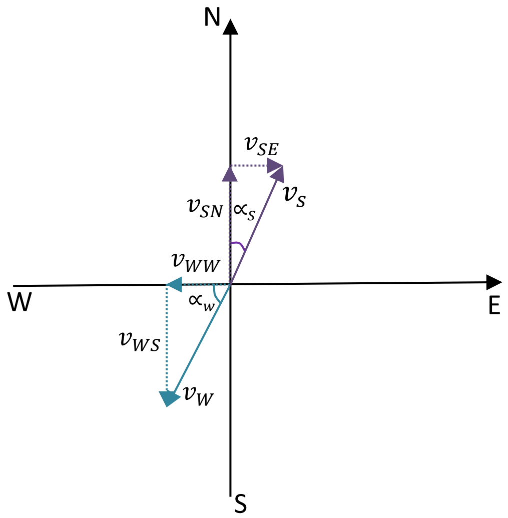
Answer on Question#61937 – Physics – Mechanics – Relativity

The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.01 m/s in a direction 25.1^0 east of north relative to the Earth. It encounters a wind that has a velocity of 4.93 m/s in a direction of 50.2^0 south of west relative to the Earth. What is the velocity of the wind relative to the water? (Enter the magnitude in m/s and the direction in degrees south of west.)

Solution. Comparing the direction on the compass and having in the Cartesian coordinate system positive direction write the component of the velocity sailboats and wind.



According to the condition of the problem: sailboat has a velocity of $v_S = 2.01\frac{m}{s}$ with $\propto_S = 25.1^0$; wind that has a velocity $v_W = 4.93\frac{m}{s}$ with $\propto_w = 50.2^0$.

Therefore the components of the velocity sailing boats in East or North direction (relative to the Earth) is equal to

$v_{SE} = v_S \cos\alpha_S = 2.01\cos 25.1^0 \approx 1.8202\frac{m}{s}$

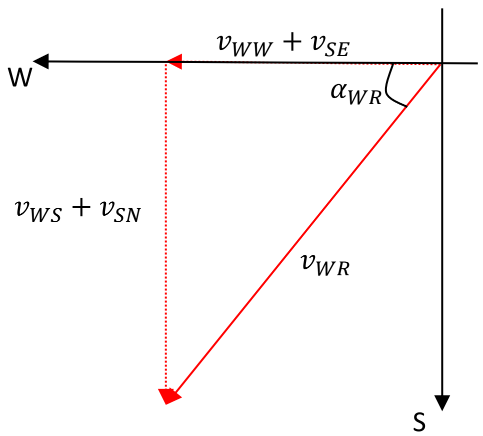
$v_{SN} = v_S \sin\alpha_S = 2.01\sin 25.1^0 \approx 0.8526\frac{m}{s}$

and the components of the wind speed in the West and South (relative to the Earth) are equal

$v_{WW} = v_W \cos\alpha_W = 4.93\cos 50.2^0 \approx 3.1557\frac{m}{s}$

$v_{WS} = v_W \sin\alpha_W = 4.93\sin 50.2^0 \approx 3.7876\frac{m}{s}$

The relative wind speed can be represented as



$v_{WW} + v_{SE} = 3.1557 + 1.8202 \approx 4.98\frac{m}{s}$

$v_{WS} + v_{SN} = 3.7876 + 0.8526 \approx 4.64\frac{m}{s}$

Hence magnitude of velocity of the wind relative to the water equal to (using the Pythagorean theorem)

$v_{WR}^2 = (v_{WW} + v_{SE})^2 + (v_{WS} + v_{SN})^2 \to v_{WR} = \sqrt{(v_{WW} + v_{SE})^2 + (v_{WS} + v_{SN})^2}$

$v_{WR} = \sqrt{(4.98)^2 + (4.64)^2} \approx 6.81\frac{m}{s}$

For a right triangle

$\tan\alpha_{WR} = \frac{v_{WS}+v_{SN}}{v_{WW}+v_{SE}} \to \alpha_{WR} = \tan^{-1}\left(\frac{v_{WS}+v_{SN}}{v_{WW}+v_{SE}}\right) = \tan^{-1}\left(\frac{4.64}{4.98}\right) \approx 43.0^0$ south of west.

Answer. $v_{WR} = 6.81\frac{m}{s}$ in a direction 43.0^0 south of west.