Answer on Question #61920-Physics-Optics

Write down Maxwell's field equations for free space and obtain expressions for (i) velocity of e.m. waves, and (ii) Poynting Vector. Also discuss its significance

Solution

$$\nabla \boldsymbol{E} = 0;\ \nabla \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t};\ \nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}.$$

Take the curl (∇×) of the curl equations:

$$\nabla \times (\nabla \times \boldsymbol{E}) = -\frac{\partial \nabla \times \boldsymbol{B}}{\partial t};\ \nabla \times (\nabla \times \boldsymbol{B}) = \frac{1}{c^2}\frac{\partial \nabla \times \boldsymbol{E}}{\partial t}.$$

Using the curl of the curl identity $\nabla \times (\nabla \times \boldsymbol{X}) = \nabla(\nabla \cdot \boldsymbol{X}) - \nabla^2 \boldsymbol{X}$ we obtain the wave equations

$$\frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} - \nabla^2 \boldsymbol{E} = 0$$

$$\frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} - \nabla^2 \boldsymbol{B} = 0.$$

i) the velocity of such waves is

$$v = c.$$

ii) Poynting Vector is

$$\boldsymbol{S} = \frac{1}{\mu_0}(\mathbf{E} \times \boldsymbol{B}).$$

The electric and magnetic field are the solutions of above wave equations:

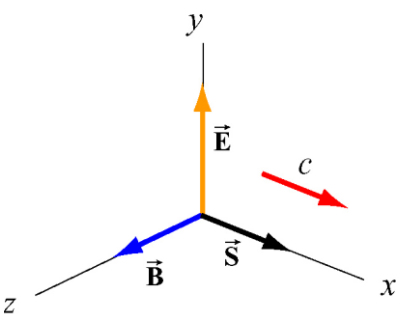
$$\boldsymbol{E} = E_0 \cos(\omega t - kx)\,\boldsymbol{j}$$

$$\boldsymbol{B} = B_0 \cos(\omega t - kx)\,\boldsymbol{k}$$

Thus,

$$\boldsymbol{S} = \frac{1}{\mu_0}(E_0 \cos(\omega t - kx)\,\boldsymbol{j} \times B_0 \cos(\omega t - kx)\,\boldsymbol{k}) = \frac{E_0 B_0}{\mu_0}\cos^2(\omega t - kx)\,\boldsymbol{i}$$

$\boldsymbol{S}$ points in the direction of wave propagation.



Poynting Vector is the rate of the energy flow per unit area. Thus, the direction of energy flow is the same as direction of wave propagation.