Answer on Question #61897, Physics / Optics

Show that the superposition of two linearly polarized light waves having different amplitude and a finite phase difference can be used to produce elliptically plane polarized light waves.

Answer:

Let a and b be the amplitudes of first linearly polarized light wave and b of second linearly polarized light wave, respectively.

If δ be the phase difference between the two emergent beams, then their vibrations can be expressed as

For first wave: $x = a\sin(\omega t + \delta)$ (1)

For second wave: $y = b\sin\omega t$ (2)

From second equation we have:

$$\frac{y}{b} = \sin\omega t$$

Hence $\cos\omega t = \sqrt{1 - \sin^2\omega t} = \sqrt{1 - \frac{y^2}{b^2}}$

From first eqation we have:

$$x = a\sin(\omega t + \delta) = a(\sin\omega t\cos\delta + \cos\omega t\sin\delta)$$

or,

$$\frac{x}{a} = \sin\omega t\cos\delta + \cos\omega t\sin\delta = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}}\sin\delta$$

or,

$$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1 - \frac{y^2}{b^2}}\sin\delta$$

Squaring and rearranging, we get:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta$$

This is the **general equation of an ellipse**.

When $\delta = \pi/2$, then $\sin\delta = 1$ and $\cos\delta = 0$, therefore

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\frac{\pi}{2} = \sin^2\frac{\pi}{2}$$

or,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

This is the **equation of an ellipse**. In this case, the emergent light is **elliptically polarized**.