Answer on Question #61883-Physics-Mechanics-Relativity

A droplet of perspiration falls off a worker who is standing on a platform 40 m above the ground. Assume throughout that the drop is spherical (volume of a sphere is equal to $\frac{4}{3}\pi r^3$ while its surface area is equal to $4\pi r^2$). Your objective is to investigate what happens to the drop as it falls down.

(a) Suppose that as the drop falls, the droplet evaporates at a constant rate. If V is the volume of the drop, this means that $\frac{dV}{dt} = -c$, where c is a positive constant. Find an expression for $\frac{dr}{dh}$, that is, the rate of change of the droplet's radius with respect to the height h above the ground. (3 marks)

(b) A more realistic model is to suppose instead that the rate of evaporation is proportional to the surface area of the drop. Find $\frac{dr}{dh}$ now. (3 marks)

(c) Is it possible, in either of these models, that the drop disappears via evaporation just before hitting the ground?

Solution

(a) The equation of vertical motion in this situation is

$$h = h_0 - \frac{gt^2}{2}.$$

$$\frac{dh}{dt} = -gt \to \frac{dt}{dh} = -\frac{1}{gt}.$$

$$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt} \to -c = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = -\frac{c}{4\pi r^2}.$$

$$\int dV = \int -cdt$$

$$V = V_0 - ct$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 - ct$$

$$r = \sqrt[3]{R^3 - \frac{3ct}{4\pi}}.$$

$$\frac{dr}{dt} = -\frac{c}{4\pi\left(R^3 - \frac{3ct}{4\pi}\right)^{\frac{2}{3}}}.$$

$$\frac{dr}{dh} = \frac{dr}{dt}\frac{dt}{dh} = -\frac{c}{4\pi\left(R^3 - \frac{3ct}{4\pi}\right)^{\frac{2}{3}}}\left(-\frac{1}{gt}\right) = \frac{c}{4\pi gt\left(R^3 - \frac{3ct}{4\pi}\right)^{\frac{2}{3}}}.$$

Using $t = \sqrt{\frac{2(h_0 - h)}{g}}$

$$\frac{dr}{dh} = \frac{c}{4\pi g\sqrt{\frac{2(h_0-h)}{g}}\left(R^3 - \frac{3c}{4\pi}\sqrt{\frac{2(h_0-h)}{g}}\right)^{\frac{2}{3}}}.$$

(b)

$$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt} \to -k4\pi r^2 = 4\pi r^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = -k.$$

$$\frac{dr}{dh} = \frac{dr}{dt}\frac{dt}{dh} = -k\left(-\frac{1}{gt}\right) = \frac{k}{gt}.$$

Using $t = \sqrt{\frac{2(h_0-h)}{g}}$

$$\frac{dr}{dh} = \frac{k}{g\sqrt{\frac{2(h_0-h)}{g}}} = \frac{k}{\sqrt{2g(h_0-h)}}.$$

(c) Yes, if the initial radius of a droplet is small enough.

1\. $r = \sqrt[3]{R^3 - \frac{3ct}{4\pi}} = 0$, when $t = \sqrt{\frac{2h_0}{g}}$:

$$R^3 = \frac{3c}{4\pi}\sqrt{\frac{2h_0}{g}}$$

$$R = \sqrt[3]{\frac{3c}{4\pi}\sqrt{\frac{2h_0}{g}}}$$

2\. $r = R - kt = 0$, when $t = \sqrt{\frac{2h_0}{g}}$:

$$R = k\sqrt{\frac{2h_0}{g}}.$$