Answer on Question #61868-Physics-Mechanics-Relativity

Derive expression for average rate at which energy is transported by a progressive wave propagating in a medium.

Solution

The total average energy of the segment of the medium under consideration at any instant of time is

$$E = \frac{1}{2} dm\, \omega^2 a^2,$$

where ω is the angular frequency and a is an amplitude.

This equation gives the total energy carried by a progressive wave and transported per cycle through a thin layer of mass dm of the medium.

$$dm = \rho A\, \Delta x,$$

where ρ is the density, A is the cross-sectional area, Δx is the thickness of the layer.

Now, we can write the expression for the power average rate at which energy is transported by a progressive wave propagating in a medium:

$$P = \frac{E}{\Delta t} = \frac{\frac{1}{2}\rho A\, \Delta x\, (2\pi f)^2 a^2}{\frac{\Delta x}{v}},$$

where we used the expression $\omega = 2\pi f$ for the frequency and $\Delta t = \frac{\Delta x}{v}$ for the time taken by the wave to cross the layer of thickness Δx by the wave propagating with velocity v. Then,

$$P = 2\pi^2 \rho A a^2 f^2 v.$$