Answer on Question #61853-Physics-Solid State Physics

Show that the probability that an energy level which is at an energy ΔE above the Fermi level is occupied is equal to the probability that an energy level ΔE below the Fermi level is not occupied.

Solution

Let the energy above the Fermi energy E_F be E_1. Then $\Delta E = E_1 - E_F$, and the probability of occupancy $f(E_1)$ of the level E_1 is given by the FD distribution function, i.e.,

$$f(E_1) = \frac{1}{1 + \exp\left(\frac{E_1 - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{\Delta E}{kT}\right)}$$

The probability of vacancy of the level E_1 is

$$1 - f(E_1) = 1 - \frac{1}{1 + \exp\left(\frac{\Delta E}{kT}\right)} = \frac{\exp\left(\frac{\Delta E}{kT}\right)}{1 + \exp\left(\frac{\Delta E}{kT}\right)}$$

The probability of occupancy of an energy level E_2 below E_F, where $E_F - E_2 = \Delta E$, is

$$f(E_2) = \frac{1}{1 + \exp\left(\frac{E_2 - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(-\frac{\Delta E}{kT}\right)} = \frac{\exp\left(\frac{\Delta E}{kT}\right)}{1 + \exp\left(\frac{\Delta E}{kT}\right)} = 1 - f(E_1).$$

which proves the desired result.