Answer on Question #61832, Physics / Other

d) Calculate acoustic impedance of air and water at STP using the data
r =1.29 kg m−3, n = 332 ms−1, r =10^3 kg m−3 air air water and 1500 ms . n = −1 water
Derive the formula used.

Solution:

How to derive a formula for and calculate the specific acoustic impedance of air and water at STP, knowing that:

The density of air at STP is 1.29 kg/m³ and the speed of sound is 332m/s;

The density of water at STP is 1000 kg/m³ and the speed of sound is 1500m/s.

Specific acoustic impedance is defined as the ratio of acoustic pressure (p) to specific flow (u) (also known as acoustic flow velocity).

$$z = \frac{p}{u}$$

We know that for sound waves traveling in a medium, the displacement of the particles in the y direction at a position x and at a time t is given by:

$$y(x,t) = A_0 \sin(kx - \omega t)$$

Where Ao is the maximum displacement (the amplitude), k is the wave number (defined as $k = 2\pi/\lambda$) and w is the angular frequency of the oscillation. It is also known that the velocity u of a particle in this sound wave and the acoustic pressure p are given by the following two equations:

$$u(x,t) = \frac{\partial y}{\partial t} = -A_0 \omega \cos(kx - \omega t)$$

$$p(x,t) = -\kappa \frac{\partial y}{\partial x} = -\kappa A_0 k \cos(kx - \omega t)$$

Here, κ is defined as the adiabatic bulk modulus (it basically measures the resistance of a substance to uniform compression). Now, substituting this results in our definition of the specific acoustic impedance, we get:

$$z = \frac{\kappa k}{\omega}$$

But it can also be shown, when studying sound waves, that

$$\kappa = \rho v^2$$

Where v is the speed at which the sound wave travels in this medium and $\rho$is the density of the substance. Finally, we have for the specific acoustic impedance, using the definition of k:

$$z = \frac{\kappa k}{\omega} = \rho v^2 \frac{k}{\omega} = \rho v^2 \frac{2\pi}{\lambda \omega} = \rho v^2 \frac{2\pi}{\lambda} \frac{T}{2\pi} = \rho v$$

Now we see that the acoustic impedance of a substance is a function of the density of the medium and the velocity at which the sound wave travels in it.

Using the values given by you, we have **for air**:

$$z_{air} = (1.29 \text{ kg m}^{-3})(332 \text{ ms}^{-1}) = 428.28 \frac{kg}{m^2 s}$$

And **for water**:

$$z_{water} = (1000 \text{ kg m}^{-3})(1500 \text{ ms}^{-1}) = 1.5 \cdot 10^6 \frac{kg}{m^2 s}$$

Answer: $z_{air} = 428.3 \frac{kg}{m^2 s}$; $z_{water} = 1.5 \cdot 10^6 \frac{kg}{m^2 s}$.