Answer on Question #61831-Physics-Other

i) All particles between any two consecutive nodes in a stationary wave are in phase. Show that they are in opposite phase with the particles between the next pair of nodes.

Solution

The instantaneous displacement of stationary wave is given by

$$\psi(x,t) = 2A \cos kx \sin \omega t$$

For any two points x_1 and x_2 between consecutive nodes, the phase is same, equal to ωt. But if $x_2 = x_1 + \frac{\lambda}{2}$, i.e. the points belong to adjacent loops (which mean that they lie between consecutive pairs of nodes). The corresponding displacements are

$\psi_1(x,t) = 2A \cos kx_1 \sin \omega t$ and $\psi_2(x,t) = 2A \cos kx_2 \sin \omega t$.

$$\psi_2(x,t) = 2A \cos k\left(x_1 + \frac{\lambda}{2}\right) \sin \omega t = 2A \cos(kx_1 + \pi) \sin \omega t = -\,2A \cos kx_1 \sin \omega t$$
$$= 2A \cos kx_1 \sin(\omega t + \pi)$$

This result shows that the phase of ψ_1 and ψ_2 differ by π.

ii) The equation of a stationary wave is given by

y(x,t) = 2sin pxcos100pt,

where x and y are in meters and t is in seconds. Calculate the amplitude, wavelength and frequency of component waves whose superposition generated stationary waves.

Solution

$$Y(x,t) = 2sin\, \pi x\, Cos 100\pi\, t$$

x and y are in meters and t is in seconds.

We use the formula in trigonometry

$$2sin\, A\, Cos\, B = sin\,(A + B) + sin\,(A - B)$$

$$y(x,t) = 2sin\, \pi x\, Cos 100\pi\, t = Sin\,(\pi x + 100\pi t) + Sin(\pi x - 100\,\pi t)$$
$$= sin\,(\pi x + 100\pi t) - sin(100\pi t - \pi x)$$
$$= sin\,(100\,\pi t + \pi x) + sin(100\,\pi t - \pi x + \pi)$$

general formula for a standing wave :

$$y\,(x,t) = A\, sin\,(\omega\, t - k\, x + \Phi)$$

These are the component waves which are part of the stationary wave:

$$y_1 = \sin(100\,\pi t + \pi x)$$

$$y_2 = sin(100\,\pi t - \pi x + \pi)$$

The amplitude is

$$A = 1\ m.$$

The wavelength is

$$\lambda = \frac{2\pi}{\pi} = 2\ m.$$

The frequency is

$$f = \frac{100\ \pi}{2\pi} = 50\ Hz.$$