Answer on Question #61825, Physics / Other

Due to a mild earthquake, a horizontal book shelf in a library moves vertically in a SHM with a period 1s. Derive expression for its acceleration amplitude and calculate the maximum amplitude that it can have so that the books resting on it continue to be in contact with it always.

Take g = 10 ms^{-2}.

Solution:

If a book shelf moves in a Simple Harmonic Motion (SHM), its motion can be described by the equation

$$y = y_0 \sin(\omega t + \varphi)$$

Here, y is the vertical displacement of the bookshelf from the equilibrium, y_0 is the amplitude (maximum displacement), ωis a angular frequency of the motion, t is time and ϕ is the phase shift, determined by initial conditions. Here, we can choose the initial conditions so that at time t = 0 the shelf is in equilibrium, which we can let to correspond to y = 0.

Then,

$$y = y_0 \sin(\omega t)$$

The angular frequency is determined by the period of SHM:

$$\omega = \frac{2\pi}{T}$$

So, if the period is T = 1 s, then ω= 2pi = 6.28 rad/s. Therefore, the equation of the SHM motion becomes

$$y = y_0 \sin(2\pi t)$$

The acceleration of the bookshelf at any time can be found by differentiating y(t) twice with respect to time. In any SHM, the acceleration and the displacement are related as

$$a = \frac{d^2y}{dt^2} = \frac{d}{dt}(2\pi y_0 \cos(2\pi t)) = -4\pi^2 y_0 \sin(2\pi t)$$

The amplitude of the acceleration is then

$$\boldsymbol{a_0 = 4\pi^2 y_0}$$

To find the condition for the books to always be in the contact with the shelf, consider the second Newton's Law for a book:

$$mg + N = ma$$

N is the normal force acting from the bookshelf on the books. If a book is no longer in contact with the shelf, N = 0.

When the acceleration of the bookshelf and the books is upward, then

$$-mg + N = ma$$

$$N = m(g + a)$$

N will always be positive, no matter what a is.

When the acceleration of the bookshelf and the books is negative, then

$$mg - N = ma$$

and

$$M = m(g - a)$$

Then, if a becomes equal or greater than g, the normal force N will become 0 and the books will lose the contact with the shelf. So, the condition for the books to remain in contact with the shelf is

$$a \leq g$$

Then, the maximum magnitude of the acceleration must be less or equal to g:

$$a_0 = 4\pi^2 y_0 \leq g$$

From here, the maximum amplitude of SHM is

$$y_0 = \frac{g}{4\pi^2}$$

$$y_0 = \frac{10}{4\pi^2} = 0.253 \text{ m}$$