Answer on Question #61803 - Physics - Mechanics | Relativity

Question:

A ball of mass m and radius R rotates with an angular velocity ω_0 about a horizontal axis. If it is placed on a horizontal plane, the coefficient of friction being μ, how far can the ball travel before it is engaged in a pure rolling motion?

Solution:

$$\vec{F} = m\vec{a};$$

$$\begin{cases} F = ma, \\ F = \mu mg; \end{cases}$$

Now we will find angular acceleration:

$$\vec{M} = I\vec{\beta} = [\vec{R}, \vec{F}];$$

$$M = I\beta = FR;$$

$$I\beta = FR;$$

$$\frac{2}{5}mR^2\beta = \mu mgR \Rightarrow \beta = \frac{5\mu g}{2R};$$

Now we will find angular velocity:

$$ma = \mu mg \Rightarrow a = \mu g \Rightarrow \frac{V}{t} = \mu g \Rightarrow \frac{\omega R}{t} = \mu g \Rightarrow \omega = \frac{\mu g t}{R};$$

Now we can find time of motion without pure rolling:

$$\omega = \omega_0 - \beta t \Rightarrow \frac{\mu g t}{R} = \omega_0 - \frac{5\mu g}{2R}t \Rightarrow t = \frac{2R\omega_0}{7\mu g};$$

And finally, we can find how far the ball can travel before it is engaged in a pure rolling motion:

$$S = \frac{at^2}{2} = \frac{4\mu g R^2 \omega_0^2}{2 \cdot 49\mu^2 g^2} = \frac{2R^2\omega_0^2}{49\mu g};$$

Answer:

$$S = \frac{2R^2\omega_0^2}{49\mu g};$$