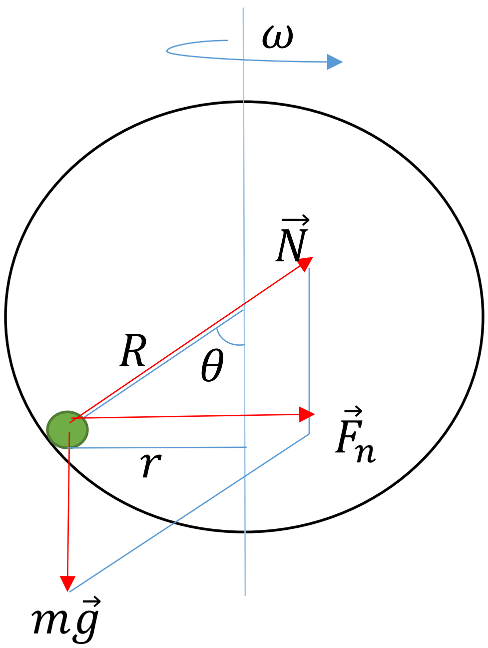
Answer on Question #61763 - Physics / Mechanics | Relativity

Question:

The small spherical balls are free to move on the inner surface of the rotating spherical chamber of radius R=2.0m. Sphere is rotating about the axes X-X' with angular velocity ω. If the balls reach a steady state at an angular position Θ=45°, the angular speed ω of device is

Solution:



The sketch of the problem is shown in figure above. Two external forces act on a ball: weight $m\vec{g}$ and reaction force of the inner surface of a sphere on vertical plane (we don't pay attention to friction, which acts on the horizontal plane, keeping ball holding on sphere). According to Newton's second law, net force of these two forces $\vec{F_n} = ma_c$, where a_c is the centripetal acceleration of the ball which is $a_c = \omega^2 r$. So $F_n = m\omega^2 r$.

On the other hand $\vec{F_n}$ can be found from tangent of the angle θ:

$$\tan\theta = \frac{F_n}{mg}$$

From where $F_n = mg\tan\theta$.

From two equation for F_n we can write:

$$mg\tan\theta = ma_c$$

$$g\tan\theta = \omega^2 r$$

$$\omega = \sqrt{\frac{g \tan\theta}{r}}.$$

r can be found from sine of the angle θ as follows:

$$\sin\theta = \frac{r}{R}$$

$$r = R\sin\theta.$$

Knowing that $\frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta}$

Final result is

$$\omega = \sqrt{\frac{g}{R\cos\theta}} = \sqrt{\frac{9.8}{2 \times \cos 45}} \approx 2.63\,\frac{\text{rad}}{\text{s}}.$$

Answer: $2.63\ \frac{\text{rad}}{\text{s}}$.