Answer on Question #61743-Physics-Other

For gravity waves, the phase velocity is given by vp = C λ1/2. Show that group velocity for these waves is half of their phase velocity

Solution

Starting from the dispersion ω(k), the phase velocity is

$$v_p = \frac{\omega}{k} = \mathrm{C}\,\lambda^{\frac{1}{2}}\ ,$$

where λ and k are the wavelength and wavenumber respectively. As we know, the relation between wavelength and wavenumber is described as follow:

$$\lambda = \frac{2\pi}{k},$$

Therefore, we can rewrite the phase velocity in a form:

$$v_p = \frac{\mathrm{C}\sqrt{2\pi}}{\mathrm{k}^{\frac{1}{2}}}.$$

Thus, we obtain

$$\omega = kv_p = k\frac{\mathrm{C}\sqrt{2\pi}}{\mathrm{k}^{\frac{1}{2}}} = \mathrm{C}\sqrt{2\pi k}$$

The group velocity is a derivative:

$$v_g = \frac{\partial\omega}{\partial k} = \frac{\partial}{\partial k}\left(\mathrm{C}\sqrt{2\pi k}\right) = \frac{1}{2}\mathrm{C}\sqrt{\frac{2\pi}{k}}.$$

We can write it as

$$v_g = \frac{1}{2}\mathrm{C}\lambda^{\frac{1}{2}} = \frac{1}{2}v_p.$$

We can see that group velocity for these waves is half of their phase velocity.

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