Answer on question #61739, Physics, Electromagnetism

Derive an expression for divergence E (divE) for a point inside and outside of a charge distribution?

Solution:

$$\vec{E} = \frac{kQ}{r^3}\vec{r} = \frac{kQ}{r^3}\left(x\vec{\imath} + y\vec{\jmath} + z\vec{k}\right)$$

Where $\vec{r}$ the radius vector of the point charge location

For a point outside of a charge distribution

$$div\vec{E} = kQ\left[\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r^3}\right)\right]$$

$$\frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) = \frac{r^3 + 3xr^2\frac{\partial r}{\partial x}}{r^6} = \frac{r^3 - 3rx^2}{r^6} = \frac{r^2 - 3x^2}{r^5}$$

Similarly

$$\frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) = \frac{r^2 - 3y^2}{r^5}$$

$$\frac{\partial}{\partial z}\left(\frac{z}{r^3}\right) = \frac{r^2 - 3z^2}{r^5}$$

Where, $|r| = \sqrt{x^2 + y^2 + z^2}$

Then,

$$div\vec{E} = kQ\frac{3r^2 - 3(x^2 + y^2 + z^2)}{r^5} = kQ\left(\frac{3}{\left(\sqrt{x^2 + y^2 + z^2}\right)^3} - \frac{3(x^2 + y^2 + z^2)}{\left(\sqrt{x^2 + y^2 + z^2}\right)^5}\right) = 0$$

Thus, if r≠0, then $div\vec{E} = 0$. At the point r = 0 divergence undefined ($div\vec{E} = \infty$). We can calculate the flow field $\vec{E}$ through the surface surrounding the charge is Q. Let there be a spherical closed surface of arbitrary radius, and the center point charge is Q.

For a point inside of a charge distribution

$$\frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_V \rho dV$$

Where, ρ bulk density of charge, by using Gauss' theorem, we can write

$$\oint_S \left(\vec{E}\vec{n}\right) dS = \int_V div\vec{E}dV$$

left sides of the formulas are

$$\oint_S \left(\vec{E}\vec{n}\right) dS = \frac{Q}{\varepsilon_0}$$

therefore, equal and right

$$\frac{1}{\varepsilon_0}\int_V \rho dV = \int_V div\vec{E}dV$$

and therefore the integrands

$$div\vec{E} = \frac{\rho}{\varepsilon_0}$$

Answer:

Divergence for a point outside of a charge distribution

$$div\vec{E} = kQ\left(\frac{3}{\left(\sqrt{x^2+y^2+z^2}\right)^3} - \frac{3(x^2+y^2+z^2)}{\left(\sqrt{x^2+y^2+z^2}\right)^5}\right) = 0$$

Divergence for a point inside of a charge distribution

$$div\vec{E} = \frac{\rho}{\varepsilon_0}$$