Answer on Question #61701-Physics-Mechanics

A satellite of mass 2500 kg is orbiting the Earth in an elliptical orbit. At the farthest point from the Earth, its altitude is 3600 km, while at the nearest point, it is 1100 km. Calculate the energy and angular momentum of the satellite and its speed at the aphelion and perihelion.

Solution

We know, $U = -\frac{GMm}{r}$ (where G IS gravitational constant, m is satellite's mass, M IS Earth's mass)

At aphelion,

$$r_1 = R_{EARTH} + 3600\ km$$

At perihelion,

$$r_2 = R_{EARTH} + 1100\ km$$

From the conservation of energy:

$$K + U = const \rightarrow \frac{mv_1^2}{2} - \frac{GMm}{r_1} = \frac{mv_2^2}{2} - \frac{GMm}{r_2}$$

From the conservation of angular momentum:

$$mv_1r_1 = mv_2r_2$$

$$v_2 = \frac{r_1}{r_2}v_1$$

$$\frac{v_1^2}{2} - \frac{GM}{r_1} = \frac{1}{2}\left(\frac{r_1}{r_2}v_1\right)^2 - \frac{GM}{r_2}$$

$$v_1^2 = GM\frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{1 - \left(\frac{r_1}{r_2}\right)^2}$$

1. Speed.

At aphelion,

$$v_1 = \sqrt{6.673 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24}\frac{\left(\frac{1}{(6.37 \cdot 10^6 + 3.6 \cdot 10^6)} - \frac{1}{(6.37 \cdot 10^6 + 1.1 \cdot 10^6)}\right)}{1 - \left(\frac{(6.37 \cdot 10^6 + 3.6 \cdot 10^6)}{(6.37 \cdot 10^6 + 1.1 \cdot 10^6)}\right)^2}} = 4140.5\frac{m}{s}$$

At perihelion,

$$v_2 = \frac{(6.37 \cdot 10^6 + 3.6 \cdot 10^6)}{(6.37 \cdot 10^6 + 1.1 \cdot 10^6)}4140.5 = 5526.2\frac{m}{s}$$

2. The energy.

At aphelion,

$$E_a = \frac{1}{2}2500(4140.5)^2 - \frac{6.673 \cdot 10^{-11} \cdot 5.98 \cdot 10^{24} \cdot 2500}{(6.37 \cdot 10^6 + 3.6 \cdot 10^6)} = -7.86 \cdot 10^{10} J.$$

At perihelion,

$$E_p = E_a = -7.86 \cdot 10^{10} J.$$

2. The angular momentum.

At aphelion,

$$L_a = 2500(4140.5)(6.37 \cdot 10^6 + 3.6 \cdot 10^6) = 1.03 \cdot 10^{14} \frac{kgm^2}{s}.$$

At perihelion,

$$L_p = L_a = 1.03 \cdot 10^{14} \frac{kgm^2}{s}.$$