Answer on Question 61693, Physics, Mechanics, Relativity

Question:

A particle of mass M is released from rest on a rough inclined plane, which makes an angle of 30° with the horizontal. It is observed that the particle moves a distance of $3\ m$ in $3\ s$. What is the particle's acceleration? Draw a properly labelled free body diagram. Calculate the coefficient of kinetic friction between the particle and the surface of the inclined plane.

Solution:

a) We can find the particle's acceleration from the kinematic equation:

$$d = v_0 t + \frac{1}{2} a t^2,$$

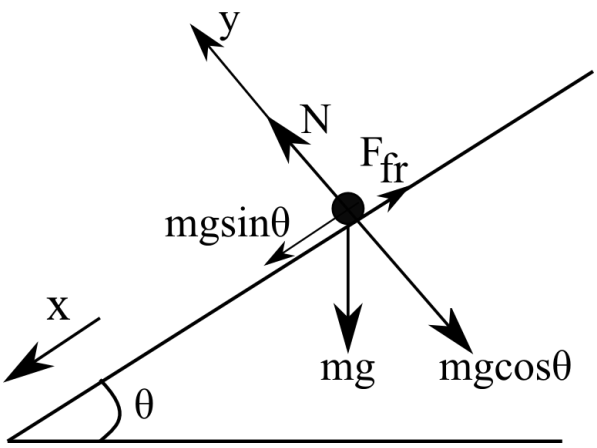
here, d is the distance, v_0 is the initial velocity of the particle (because the particle is released from rest $v_0 = 0\ ms^{-1}$), t is the time during which the particle moved the distance d and a is the particle's acceleration which we are searching for.

Then, from this formula we can find the particle's acceleration:

$$d = \frac{1}{2} a t^2,$$

$$a = \frac{2d}{t^2} = \frac{2 \cdot 3\ m}{(3\ s)^2} = 0.67\ \frac{m}{s^2}.$$

b) There are three forces that act on the particle: the force of gravity Mg directed downward and can be resolved into two perpendicular components ($F_{\parallel} = Mgsin\theta$ and $F_{\perp} = Mgcos\theta$), the force of reaction directed perpendicular to the surface of the inclined plane and the friction force F_{fr} directed opposite to the motion of the particle. Let's draw a free-body diagram and write all the forces that act on the particle:



$$M\vec{g} + \vec{N} + \overrightarrow{F_{fr}} = m\vec{a}.$$

Then projected the forces on axis x and y we get:

$$Mgsin\theta - F_{fr} = Ma, \ (1)$$

$$N - Mgcos\theta = 0. \ (2)$$

Let's find the friction force that acts on the particle:

$$F_{fr} = \mu_k N = \mu_k Mgcos\theta.$$

Substituting the friction force into the first equation we get:

$$Mgsin\theta - \mu_k Mgcos\theta = Ma,$$

$$gsin\theta - \mu_k gcos\theta = a.$$

From the last equation we can find the coefficient of kinetic friction between the particle and the surface of the inclined plane:

$$\mu_k = \frac{gsin\theta - a}{gcos\theta} = \frac{9.8\ \frac{m}{s^2} \cdot sin30^\circ - 0.67\ \frac{m}{s^2}}{9.8\ \frac{m}{s^2} \cdot cos30^\circ} = 0.5.$$

Answer:

a) $a = 0.67\ \frac{m}{s^2}$.

b) $\mu_k = 0.5$.