Answer on Question #61614 - Physics - Mechanics | Relativity

## Question:

Consider N identical masses connected through identical springs of force constant k. The free ends of the coupled system are rigidly fixed at x = 0 and x = l. The masses are made to execute longitudinal oscillations on a frictionless table.
1) Depict the equilibrium as well as instantaneous configurations.
2) Write down their equations of motion, decouple them and obtain frequencies of normal modes.

## Answer:

1) Suppose the length of each spring in its natural unextended or uncompressed form is a. Then the total length of N+1 springs is (N+1)a. In the equilibrium position, the springs are not oscillating.

Mass1 is at x = a.
Mass 2 is at x = 2a.
Mass N is at x = Na.

On either side of each mass the force acting on it is: k Δ

Each is extended by:

$$\Delta = \frac{L}{N+1} - a$$

Mass1 is at x = a + Δ.
Mass 2 is at x = 2 a + 2Δ.
Mass N is at x = N a + NΔ.

2) Equations of motion of the masses are:

$$m\frac{d^2x_1}{dt^2} = -k(2x_1 - x_2)$$

$$m\frac{d^2x_2}{dt^2} = -k(2x_2 - x_1 - x_3)$$

$$m\frac{d^2x_3}{dt^2} = -k(2x_3 - x_2 - x_4)$$

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$$m\frac{d^2x_{n-1}}{dt^2} = -k(2x_{n-1} - x_{n-2} - x_n)$$

$$m\frac{d^2x_n}{dt^2} = -k(2x_n - x_{n-1})$$

Suppose that: $x_n(t) = A_n \cos(\omega t) \Rightarrow \frac{d^2x_n}{dt^2} = -\omega^2 x_n(t)$

Let $2 - \frac{m\omega^2}{k} = 2 - \frac{\omega}{\omega_0} = C$

We get equations like:

$$Cx_1 = x_2$$

$Cx_2 = x_1 + x_3$

$Cx_3 = x_2 + x_4$

………………………….

$Cx_{n-1} = x_{n-2} + x_n$

$Cx_n = x_{n-1}$

The system can be solved knowing the number of masses in it:

For 3 masses we have:

$$C = \pm\sqrt{2}\,, \frac{\omega}{\omega_0} = 2 \pm \sqrt{2}$$