Answer on Question #61581 - Physics - Mechanics | Relativity

Question:

Establish the equation of motion of a damped oscillator. Solve it for a weakly damped oscillator and discuss the significance of the results.

Answer:

The equation of motion of a damped oscillator is:

$$m\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + kx = 0$$

or

$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0$, where $2\beta = \frac{\alpha}{m}$ and $\omega_0^2 = \frac{k}{m}$.

Now we can solve it:

$$\lambda^2 + 2\beta\lambda + \omega_0^2 = 0 \Rightarrow \lambda_1 = -\beta + \sqrt{\beta^2 - \omega_0^2},\ \lambda_2 = -\beta - \sqrt{\beta^2 - \omega_0^2}$$

Oscillator will be a weakly damped only when $\beta < \omega_0$:

$\lambda_1 = -\beta + i\omega_\beta$, $\lambda_2 = -\beta - i\omega_\beta$ where $\omega_\beta = \sqrt{\omega_0^2 - \beta^2}$

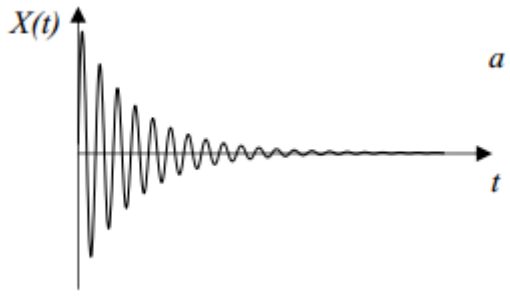
And now we have:

$$x(t) = A_1 e^{(-\beta + i\omega_\beta)t} + A_2 e^{(-\beta - i\omega_\beta)t}$$

$$x(t) = e^{-\beta t}(A_1 e^{i\omega_\beta t} + A_2 e^{-i\omega_\beta t})$$

$$x(t) = Ae^{-\beta t}\cos(\omega_\beta t + \varphi)$$

$Ae^{-\beta t}$ is the amplitude that decreases with time.



This solution of equation of motion of a damped oscillator describes the majority of oscillations in nature, such as mathematical pendulum movement in the air or in the water.