Answer on question #61328, Physics, Electromagnetism

7) What are the dimensions of the constant k in Coulomb's law of electrostatics?

a) $ML^2T^{-4}T^{-2}A^{-1}$

b) $ML^2T^3A^{-2}$

c) $M^{-2}L^3T^2A^{-1}$

d) $ML^3T^{-4}A^{-2}$

Solution:

Coulomb's law is

$$F = k\frac{q_1 q_2}{r^2}$$

Therefore, k equals:

$$k = \frac{Fr^2}{q_1 q_2}$$

Dimension of force is N, dimension of distance is m, dimension of charge is C.

Dimension of N is

$$[N] = \left[\frac{kg \cdot m}{s^2}\right]$$

Dimension of C is

$$[C] = [A \cdot s]$$

$$[k] = \left[\frac{kg \cdot m}{s^2} \cdot \frac{m^2}{A^2 \cdot s^2}\right] = \left[\frac{kgm^3}{s^4 A^2}\right] = ML^3T^{-4}A^{-2}$$

Answer: d) $ML^3T^{-4}A^{-2}$

8) The two tiny balls shown in the figure have identical masses of 0.20 g each. When suspended from 50-m long string, they make an angle of 37° to the vertical. If the charges on each ball are the same, how large is each charge? Click here to see exhibit

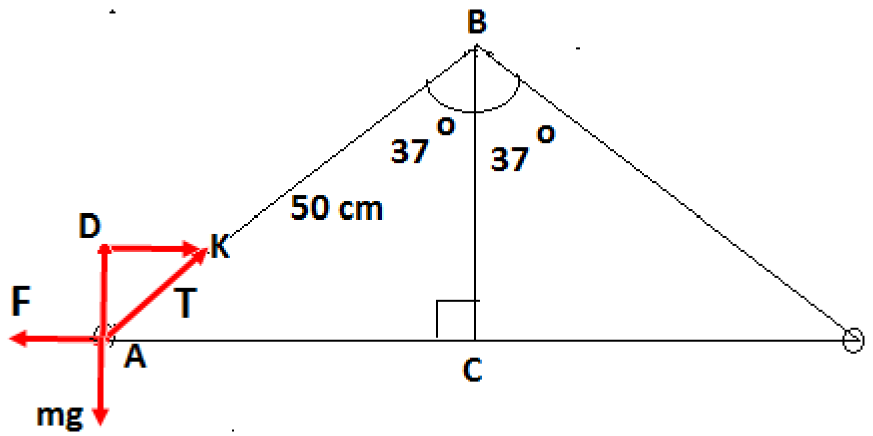
a) 0.48μC

b) 0.36μC

c) 0.24μC

d) 0.62μC

Solution:



The sum of the internal angles of a triangle is always 180°.

So,

$$\angle A + \angle B + \angle C = 180°$$

$$\angle B = 90°,\ \angle C = 37°,\ \angle A = ?$$

$$\angle A = 180° - 90° - 37° = 53°$$

Angles ∠BAC and ∠DAK, we can take as complementary angles. They form a right angle in the amount of 90°.

$$\angle BAC + \angle DAK = 90°$$

$$\angle DAK = 90° - 53° = 37°$$

Since the system is at rest, we can apply the conditions for equilibrium to the ball on the left. Three force act on the ball: its weight mg, the tension T in the string, and F, the repulsive forces due to the charge on the other ball.

With the triangle KDA find DK and DA

$$DK = AK \cdot \sin \angle DAK$$

$$AK = T$$

$$DK = T\sin 37^{\circ} = \mathbf{0.6T}$$

$$DA = AK \cdot \sin \angle DKA$$

$$DA = T\sin 53^{\circ} = \mathbf{0.8T}$$

Conditions for equilibrium:

$$\sum F_x = 0$$

Therefore,

$$F - 0.6T = 0$$

$$\sum F_y = 0$$

Which gives

$$0.8T - mg = 0$$

$$T = \frac{mg}{0.8} = \frac{0.2 \cdot 10^{-3} kg \times 9.8\, {}^{m}/{}_{s^2}}{0.8} = 2.45 \cdot 10^{-3} N$$

Find F,

$$F = 0.6 \times 2.45 \cdot 10^{-3} = 1.47 \cdot 10^{-3} N$$

Coulomb's law is

$$F = k\frac{q_1 q_2}{r^2}$$

Find r/2, with the triangle ACB (r/2 = AC)

$$AC = AB \cdot \sin \angle ABC$$

$$AC = 50\ cm \sin 37^{\circ} = \mathbf{30\ cm}$$

r=2·30cm = 60 cm= 0.6 m

$$1.47 \cdot 10^{-3} = 9 \cdot 10^{9} \frac{q^2}{0.6^2}$$

$$q = \sqrt{\frac{1.47 \cdot 10^{-3} \times 0.6^2}{9 \cdot 10^{9}}} = 2.4 \cdot 10^{-7} C = 0.24\ \mu C$$

Answer: c) 0.24μC