Question:

1. IF a small planet were discovered whose orbital period was twice that of Earth, how many time farther from the Sun would this planet be?
2. Determine Kepler's constant for any satellite of Earth.

Solution:

1.

The third Kepler's law is: $\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3} \Rightarrow \frac{a_1}{a_2} = \sqrt[3]{\frac{T_1^2}{T_2^2}},$$

where a_2 is the radius of Earth's orbit and a_1 is the radius of planet's orbit, T_2 and T_1 are orbital periods of Earth and planet.

$$\frac{a_1}{a_2} = \sqrt[3]{\frac{T_1^2}{T_2^2}} = \sqrt[3]{\frac{(2T_2)^2}{T_2^2}} = 4^{\frac{1}{3}} \approx 1.5874 \ldots$$

2.

Kepler's constant is: $K = \frac{a^3}{T^2} = \frac{GM}{4\pi^2}$

For Earth: $K = \frac{GM}{4\pi^2} = \frac{6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}{4\pi^2} \approx 1.014 \cdot 10^{13}$ $\left(\frac{m^3}{s^2}\right)$

Answer:

1) $\frac{a_1}{a_2} \approx 1.5874 \ldots$

2) $K \approx 1.014 \cdot 10^{13}$