Answer on Question #61254-Physics-Quantum Mechanics

The quantum mechanical wave function for a particle is given by

$$\psi\,(x) = \begin{cases} Ax^{\frac{3}{2}}\ e^{-ax}, x > 0 \\ 0\,, x < 0 \end{cases}$$

Determine (i) the normalization constant A and (ii) the most probable position.

Solution

(i)

$$1 = \int_{-\infty}^{\infty} |\psi\,(x)|^2 dx = \int_{0}^{\infty} A^2 x^3\ e^{-2ax} dx = A^2 \int_{0}^{\infty} x^3\ e^{-2ax} dx = |y = 2ax| = \frac{A^2}{(2a)^4} \int_{0}^{\infty} y^3\ e^{-y} dy$$

$$= \frac{A^2}{(2a)^4} \Gamma(4) = \frac{A^2}{(2a)^4} 3! = \frac{3A^2}{8(a)^4}$$

$$A = 2\sqrt{\frac{2}{3}} a^2$$

(ii)

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^*(x) x \psi\,(x) dx = \int_{0}^{\infty} A^2 x^4\ e^{-2ax} dx = \int_{0}^{\infty} \frac{8a^4}{3} x^4\ e^{-2ax} dx = |y = 2ax| = \frac{1}{12a} \int_{0}^{\infty} y^4\ e^{-y} dy$$

$$= \frac{1}{12a} \Gamma(5) = \frac{1}{12a} 4! = \frac{2}{a}.$$