Answer on Question #61192, Physics / Mechanics | Relativity

A train travels between two stations ½ mile apart in a minimum time of 41 sec. If the train accelerates and decelerates at 8 ft/sec^2, starting from rest at the first station and coming to a stop at the end of the station, what is its maximum speed in mph? how long does it travel at this top speed?

Solution:

The mile is an English unit of length of linear measure equal to 5,280 feet.

So, the halfway between two stations is

$$d_1 = \frac{1}{4}\ mile = \frac{5280\ ft}{4} = 1320\ ft$$

Let's say that the train takes t_1 time to reach the max. speed v and then it travels at this top speed distance d_2 at time t_2.

Use the kinematic equation

$$d_1 = \frac{at_1^2}{2} + \frac{vt_2}{2}$$

The time is

$$t_1 + \frac{t_2}{2} = \frac{t}{2} = \frac{41\ s}{2} = 20.5\ \text{s}$$

The equation for speed is

$$v = at_1$$

Thus, substituting in first equation

$$d_1 = \frac{at_1^2}{2} + \frac{at_1t_2}{2}$$

$$1320 = \frac{8t_1^2}{2} + \frac{8t_1(41 - 2t_1)}{2}$$

$$330 = t_1^2 + 41t_1 - 2t_1^2$$

$$t_1^2 - 41t_1 + 330 = 0$$

$$(t_1 - 30)(t_1 - 11) = 0$$

The physical solution is

$$t_1 = 11\ s$$

Hence,

$$v = at_1 = 8 \cdot 11 = 88\ ft/s$$

1 Foot per Second = 0.681818 Miles per Hour

Thus,

$$v = 88 \cdot 0.681818 = 60\ mph$$

The distance that it travels at this top speed is

$$d_2 = vt_2 = v(41 - 2t_1) = 88(41 - 22) = 1672\ ft$$

Answer: $60\ mph; 1672\ ft.$