Question #61173, Physics / Solid State Physics

Using the expression of Bose-Einstein distribution function for photons derive Plancks Law and show that (i) Rayleigh-Jeans Law, and (ii) Wien’s Law follow from it at low & high frequencies .

The answer to the question.

$$< n >_{B-E} = \frac{1}{exp\left(\frac{E}{kT}\right)-1}.$$

Consider radiation located within the closed cavity, whose walls are heated to a certain temperature T. As noted above, this radiation is an ideal gas photons. The energy distribution of the gas particles is described by the expression $< \mathbf{n} >_{\mathbf{B-E}} = \frac{\mathbf{1}}{\mathbf{exp}\left(\frac{\mathbf{E}}{\mathbf{kT}}\right)-\mathbf{1}}$. We find the energy of the radiation in a narrow energy range from **E** to **dE**. This energy is the sum of the energies of individual photons. The density of quantum states **g(E).** Произведение на дает число квантовых состояний, заключенных внутри интервала . Multiplication **g(E)** on **dE** the giving the number of quantum states, prisoners within the interva **dE**.

Multiplying this multiplication by the average number of photons in the state $< \mathbf{n} >^{\mathbf{fot}}$ and the energy of the photon **E**, we find that the total energy of the photons in the interval **dE** is

$< \mathbf{n} >^{\mathbf{fot}} \mathbf{g}(\mathbf{E})\mathbf{EdE}$.

Let us now consider the frequency range corresponding to this energy range, ie, frequency range from $\boldsymbol{\omega} = \frac{\mathbf{E}}{\hbar}$ **to** $\boldsymbol{\omega} + \mathbf{d}\boldsymbol{\omega} = \frac{\mathbf{E}}{\hbar} + \frac{\mathbf{dE}}{\hbar}$.

We derive an expression for the same energy with the help of bulk spectral energy density $\mathbf{u_0}$.

The photon energy in the frequency range $\mathrm{d}\omega$ is $\mathrm{u_0 V d}\omega$, where V - the volume of the cavity. Equating these two expressions, we get $\boldsymbol{u_0 V}\mathbf{d}\boldsymbol{\omega} = < \mathbf{n} >^{\boldsymbol{phot}} \mathbf{g}(\mathbf{E})\mathbf{EdE}$.

Taking into account all the relations we obtain Planck's formula

$$\boldsymbol{u_0} = \frac{\boldsymbol{\hbar\omega^3}}{\boldsymbol{\pi^2 c^3}} \frac{\mathbf{1}}{\boldsymbol{exp}\left(\frac{\boldsymbol{\hbar\omega}}{\boldsymbol{kT}}\right) - \mathbf{1}}.$$

(i) Rayleigh-Jeans Law just for the long-wavelength region of the spectrum, and adequately describes the nature of the radiation. Explain the fact of such compliance can only be using quantum mechanical approach, according to which the radiation is emitted in discrete steps. Based on Plancks Law of quantum can be obtained, which will coincide with Rayleigh-Jeans Law with $\frac{\boldsymbol{\hbar\omega}}{\boldsymbol{kT}} \ll \mathbf{1}$.

This fact is a great illustration of the validity of the principle.

(ii) Experience shows that the second Wien’s Law $u_W = C_1 v^3 e^{-C_2 \frac{v}{T}}$ valid only in the limit of high frequencies (short wavelengths). It is a special case of the first specific Wien’s Law. Later, Max Planck showed that the second law of guilt follows from Planck's law for high-energy rays, and also found the constants *C1* and *C2*. With this in mind, the second Wien’s Law can be written as:

$$u_W = \frac{2\pi h\nu^3}{c^3} exp^{-\frac{h\nu}{kT}}$$

where h - Planck constant, k - is Boltzmann's constant, c - velocity of light in vacuum.