Answer on Question 60979, Physics, Other

Question:

A spring of negligible mass has force constant $k = 1600\ N/m$.

a) How far must the spring be compressed for $3.2\ J$ of potential energy to be stored in it?

b) You place the spring vertically with one end on the floor. You then drop a $1.2\ kg$ book onto it from a height of $0.8\ m$ above the top of the spring. Find the maximum distance the spring will be compressed.

Solution:

a) We can find the compression of the spring from the formula for the potential energy stored in the spring:

$$PE_{spring} = \frac{1}{2}k(\Delta x)^2,$$

here, k is the force constant, Δx is the compression of the spring.

From this formula we can find the compression of the spring, Δx:

$$\Delta x = \sqrt{\frac{2PE_{spring}}{k}} = \sqrt{\frac{2 \cdot 3.2\ J}{1600\ \frac{N}{m}}} = 0.063\ m = 6.3\ cm.$$

b) Let's use the Law of Conservation of Energy (the energy of the book that transfers to the spring is the change in the gravitational potential energy of the book):

$$PE_{spring\ (initial)} + PE_{gravitational\ (initial)} =$$
$$= PE_{spring\ (final)} + PE_{gravitational\ (final)}.$$

Initially, the spring is uncompressed, so $PE_{spring\ (initial)} = 0\ J$. Thus, we get:

$$PE_{spring\ (final)} = PE_{gravitational\ (initial)} - PE_{gravitational\ (final)},$$

here, $PE_{spring\ (final)}$ is the final potential energy that is stored in the spring when the book was fall onto it, $PE_{gravitational\ (initial)}$ is the initial gravitational energy of the book at the height h, $PE_{gravitational\ (final)}$ is the final gravitational energy of the book when the spring is compressed by the maximum distance Δx.

Then, we can write:

$$\frac{1}{2}k(\Delta x)^2 = mgh - mg(-\Delta x),$$

$$\frac{1}{2}k(\Delta x)^2 - mg\Delta x - mgh = 0.$$

As we can see, we obtain the quadratic equation. Let's substitute the numbers:

$$800(\Delta x)^2 - 11.76\Delta x - 9.408 = 0.$$

This quadratic equation has two roots:

$$\Delta x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{11.76 + \sqrt{(-11.76)^2 - 4 \cdot 800 \cdot (-9.408)}}{2 \cdot 800} = 0.116.$$

$$\Delta x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{11.76 - \sqrt{(-11.76)^2 - 4 \cdot 800 \cdot (-9.408)}}{2 \cdot 800} = -0.101.$$

The maximum distance can't be negative, so the correct answer will be:

$$\Delta x = 0.116\ m \approx 0.12\ m = 12\ cm.$$

Answer:

a) $\Delta x = 6.3\ cm.$

b) $\Delta x = 12\ cm.$