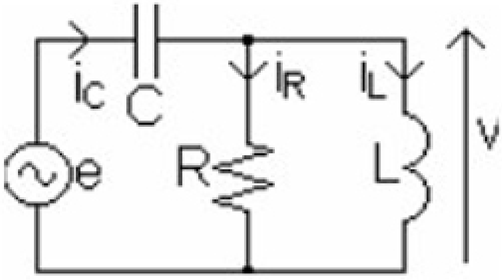
Answer on Question#60929 – Physics – Electric Circuits

Consider the circuit in Fig. P6.4 where $R = 100\Omega$, $L = 100mH$, $C = 10\mu F$, $e = 100\sin(\omega t)$ volts. Find $i_R(t)$, $i_L(t)$, $i_C(t)$, $V(t)$, the storage energy of the capacitor, the storage energy of the inductor, and the total storage energy in 3 cases:

a) $\omega = 500$rad/s, b) $\omega = 1000$rad/s, c) $\omega = 2000$rad/s



Solution. Find inductive and capacitive reactance

$X_L = \omega L$ inductive reactance, $X_C = \frac{1}{\omega C}$ – capacitive reactance. Hence

a) $X_L = 500 \cdot 0.1 = 50\Omega \quad X_C = \frac{1}{500\cdot 10\cdot 10^{-6}} = 200\Omega$

b) $X_L = 1000 \cdot 0.1 = 100\Omega \quad X_C = \frac{1}{1000\cdot 10\cdot 10^{-6}} = 100\Omega$

c) $X_L = 2000 \cdot 0.1 = 200\Omega \quad X_C = \frac{1}{2000\cdot 10\cdot 10^{-6}} = 50\Omega$

Using complex impedance is an important technique for handling multi-component AC circuits. Represent inductive reactance as $j\omega L$. R and L in parallel hence

$$\frac{1}{Z_{||}} = \frac{1}{R} + \frac{1}{j\omega L} \rightarrow Z_{||} = \frac{j\omega RL}{R + j\omega L} = \frac{j\omega LR^2 + \omega^2 L^2 R}{R^2 + \omega^2 L^2}.$$

$Z_{||}$ and C in series hence total impedance equal

$$Z = Z_{||} + \frac{j}{\omega C}.$$

$$Z = \frac{j\omega LR^2 + \omega^2 L^2 R}{R^2 + \omega^2 L^2} + \frac{j}{\omega C} = \frac{\omega^2 L^2 R}{R^2 + \omega^2 L^2} + j\left(\frac{1}{\omega C} + \frac{\omega LR^2}{R^2 + \omega^2 L^2}\right).$$

$$reZ = \frac{\omega^2 L^2 R}{R^2 + \omega^2 L^2}\ imZ = \left(\frac{1}{\omega C} + \frac{\omega LR^2}{R^2 + \omega^2 L^2}\right)$$

$$|Z| = \sqrt{(reZ)^2 + (imZ)^2}$$

a) $reZ = \frac{2500\cdot 100}{10000 + 2500} = 20\Omega\ imZ = \left(200 + \frac{50\cdot 10000}{10000 + 2500}\right) = 240\Omega\ |Z| \approx 241\Omega$

$$i_m = \frac{e_m}{|Z|} = \frac{100}{141} = 0.41A$$

Hence $i_C(t) = 0.41\sin\left(\omega t + \frac{\pi}{2}\right)$, current on capacitor. Current Triangle for a Parallel RLC Circuit

$$i_C^2 = i_R^2 + i_L^2$$

R and L in parallel hence $|i_R|R = |i_L|\omega L \rightarrow 2|i_R| = |i_L| \rightarrow |i_R| = \frac{0.41}{\sqrt{5}} \approx 0.183A$

$|i_L| = 0.366A$

$Therefore\ \ i_R(t) = 0.183\sin(\omega t)$ and $i_L(t) = 0.366\sin\left(\omega t - \frac{\pi}{2}\right)$

Storage energy of the capacitor $w = \frac{CV^2}{2} = \frac{10^{-5}\cdot(0.41\cdot 200)^2}{2} = 0.034J = 34mJ$

Storage energy of the inductor $w = \frac{LI^2}{2} = \frac{0.1\cdot 0.033489}{2} \approx 0.02J = 20mJ$

Total storage energy equal sum storage energy of the capacitor and storage energy of the inductor $w = 54mJ$.

b) $reZ = \frac{10000\cdot100}{10000+10000} = 50\Omega\ imZ = \left(100 + \frac{100\cdot10000}{10000+10000}\right) = 150\Omega\ |Z| = 158\Omega$

$$i_m = \frac{e_m}{|Z|} = \frac{100}{158} = 0.63A$$

Hence $i_C(t) = 0.63\sin\left(\omega t + \frac{\pi}{2}\right)$, max current on capacitor. Current Triangle for a Parallel RLC Circuit

$$i_C^2 = i_R^2 + i_L^2$$

R and L in parallel hence $|i_R|R = |i_L|\omega L \rightarrow |i_R| = |i_L| \rightarrow |i_R| = |i_L| = \frac{0.63}{\sqrt{2}} \approx 0.445$

$Therefore\ \ i_R(t) = 0.445\sin(\omega t)$ and $i_L(t) = 0.445\sin\left(\omega t - \frac{\pi}{2}\right)$

Storage energy of the capacitor $w = \frac{CV^2}{2} = \frac{10^{-5}\cdot(0.63\cdot100)^2}{2} \approx 0.02J = 20mJ$

Storage energy of the inductor $w = \frac{LI^2}{2} = \frac{0.1\cdot0.0198}{2} = 0.001J = 1mJ$

Total storage energy equal sum storage energy of the capacitor and storage energy of the inductor $w = 21mJ$.

c) $reZ = \frac{40000\cdot100}{40000+10000} = 80\Omega\ imZ = \left(50 + \frac{200\cdot10000}{10000+40000}\right) = 90\Omega\ |Z| = 120.4\Omega$

$$i_m = \frac{e_m}{|Z|} = \frac{100}{120.4} = 0.83A$$

Hence $i_C(t) = 0.83\sin\left(\omega t + \frac{\pi}{2}\right)$, max current on capacitor. Current Triangle for a Parallel RLC Circuit

$$i_C^2 = i_R^2 + i_L^2$$

R and L in parallel hence $|i_R|R = |i_L|\omega L \rightarrow |i_R| = 2|i_L| \rightarrow |i_L| = \frac{0.83}{\sqrt{5}} \approx 0.371A$

$|i_R| = 0.742A$

$Therefore\ \ i_R(t) = 0.742\sin(\omega t)$ and $i_L(t) = 0.371\sin\left(\omega t - \frac{\pi}{2}\right)$

Storage energy of the capacitor $w = \frac{CV^2}{2} = \frac{10^{-5}\cdot(0.83\cdot50)^2}{2} \approx 0.009J = 9mJ$

Storage energy of the inductor $w = \frac{LI^2}{2} = \frac{0.1\cdot0.137641}{2} = 0.007J = 7mJ$

Total storage energy equal sum storage energy of the capacitor and storage energy of the inductor $w = 16mJ$.