Answer on question #60875 – Physics / Atomic and Nuclear Physics

Write down the wave functions (i) ψ210 and (ii) ψ300 for the hydrogen atom. Obtain the expectation value of r for the ground state hydrogen atom.

Solution

$$\psi_{nlm}(r,\vartheta,\varphi)=\frac{1}{\sqrt{2n(n-l-1)!(n+l)!}}\left(\frac{2}{na_0}\right)^{\frac{3}{2}}\exp\left(-\frac{r}{na_0}\right)\left(\frac{2r}{na_0}\right)^{l}L_{n-l-1}^{2l+1}\left(\frac{2r}{na_0}\right)Y_{lm}(\vartheta,\varphi),$$

Where

a_0 is the Bohr radius,

L_{n-l-1}^{2l+1} is a generalized Laguerre polynomial of degree *n* − *ℓ* − 1,

$Y_{lm}(\vartheta,\varphi)$ is a spherical harmonic function of degree *ℓ* and order *m*.

Then

$$\psi_{210}(r,\vartheta,\varphi)=\frac{1}{\sqrt{24}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}}\exp\left(-\frac{r}{2a_0}\right)\left(\frac{r}{a_0}\right)\quad L_0^3\left(\frac{r}{a_0}\right)Y_{10}(\vartheta,\varphi)$$
$$=\boldsymbol{\frac{1}{4}\sqrt{\frac{1}{2\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}}\exp\left(-\frac{r}{2a_0}\right)\left(\frac{r}{a_0}\right)}$$

$$\psi_{300}(r,\vartheta,\varphi)=\frac{1}{\sqrt{72}}\left(\frac{2}{3a_0}\right)^{\frac{3}{2}}\exp\left(-\frac{r}{3a_0}\right)L_2^1\left(\frac{2r}{3a_0}\right)Y_{00}(\vartheta,\varphi)$$
$$=\boldsymbol{\frac{1}{12}\sqrt{\frac{3}{2\pi}}\left(\frac{2}{3a_0}\right)^{\frac{3}{2}}\exp\left(-\frac{r}{3a_0}\right)\left(\frac{1}{2}\left(\frac{2r}{3a_0}\right)^2-6\left(\frac{2r}{3a_0}\right)+3\right)cos\vartheta}$$

For ground state we have:

$$\psi_{100}(r,\vartheta,\varphi)=\sqrt{\frac{1}{\pi a_0^3}}\exp\left(-\frac{r}{a_0}\right)$$

The expectation value of r is:

$$\int_V r|\psi_{100}|^2\,dV=\frac{4\pi}{\pi a_0^3}\int_0^\infty r^3\exp\left(-\frac{2r}{a_0}\right)dr=\frac{4}{a_0^3}\left(\frac{a_0}{2}\right)^4\int_0^\infty x^3\exp(-x)dx=\boldsymbol{\frac{3}{2}a_0}$$