Answer on Question 60858, Physics, Mechanics | Relativity

Question:

A crate of mass $12.0\ kg$ is pulled up a rough incline with an initial speed of $2.50\ ms^{-1}$. The pulling force is $150\ N$ parallel to the incline, which makes an angle of 20° with the horizontal. The coefficient of kinetic friction is 0.3, and the crate is pulled $5.0\ m$.

1) Draw the free body diagram of the crate.

2) Determine the normal force acting on the crate.

3) Determine the magnitude of the friction force acting on the crate.

4) Determine the work done by the gravitational force on the crate.

5) Determine the increase in internal energy of the crate–incline system owing to friction.

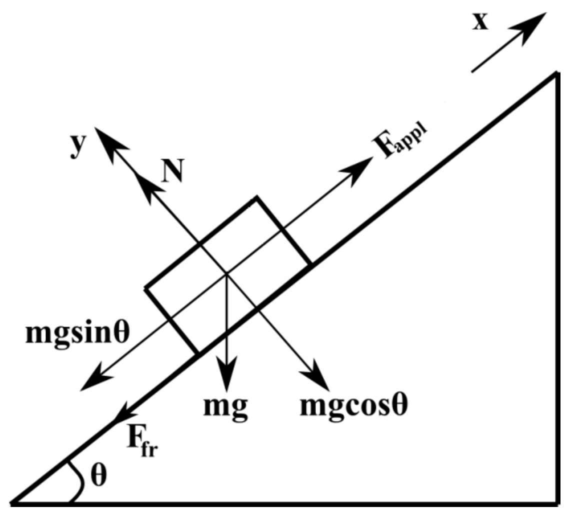
6) Determine the work done by the $150\ N$ force acting on the crate.

7) Compute the change in kinetic energy of the crate.

8) Compute the speed of the crate after being pulled $5.0\ m$ up the incline.

Solution:

1) There are four forces that act on the crate: the force of gravity mg directed downward and can be resolved into two perpendicular components ($F_{\parallel} = mgsin\theta$ and $F_{\perp} = mgcos\theta$), the force of reaction directed perpendicular to the surface, the applied (pulling) force directed parallel and upward to the slope and friction force F_{fr} directed opposite to the applied force. Let's draw a free-body diagram and write all forces that act on the crate:



2) We can find the normal force acting on the crate from the Newton's Second Law of Motion (using the projection of the forces on the y-axis):

$$\sum F_y = ma_y = 0,$$

$$N - mgcos\theta = 0,$$

$$N = mgcos\theta = 12.0\ kg \cdot 9.8\ ms^{-2} \cdot cos20^{\circ} = 110.5\ N.$$

3) We can find the magnitude of the friction force acting on the crate as follows:

$$F_{fr} = \mu_k N,$$

here, μ_k is the coefficient of kinetic friction, N is the normal force acting on the crate.

Then, we get:

$$F_{fr} = \mu_k N = \mu_k mgcos\theta = 0.3 \cdot 12.0\ kg \cdot 9.8\ ms^{-2} \cdot cos20^{\circ} = 33.15\ N.$$

4) We can find the work done by the gravitational force on the crate from the formula:

$$W_g = -mgh,$$

here, h is the height of the incline.

We can find h from the geometry:

$$sin\theta = \frac{h}{s},$$

$$h = ssin\theta,$$

here, s is the displacement of the crate.

Then, W_g will be:

$$W_g = -mgh = -mgssin\theta = -12.0\ kg \cdot 9.8\ ms^{-2} \cdot 5.0\ m \cdot sin20^{\circ} = -201.1\ J.$$

5) In section 3 we find the magnitude of the friction force acting on the crate:

$$F_{fr} = 33.15\ N.$$

Then, the increase in internal energy of the crate–incline system due to friction will be:

$$\Delta E_{int} = W_{fr} = F_{fr}s = 33.15\ N \cdot 5.0\ m = 165.75\ J.$$

6) By the definition of the work done we have:

$$W_{appl} = F_{appl}s = 150\ N \cdot 5.0\ m = 750\ J.$$

7) The change in the kinetic energy of the crate is equal to:

$$\Delta KE = W_{appl} - W_{fr} - W_g,$$

here, W_{appl} is the work done by the in-line pulling force, W_{fr} is the work done by the friction force and W_g is the work done by the gravitational force.

Then, ΔKE will be:

$$\Delta KE = W_{appl} - W_{fr} - W_g = 750\ J - 165.75\ J - 201.1\ J = 383.15\ J.$$

8) We can find the speed of the crate after being pulled 5.0 m up the incline from the formula for the change in the kinetic energy of the crate:

$$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2.$$

We already know ΔKE from the previous section. Then, v_f will be:

$$\frac{1}{2}mv_f^2 = \Delta KE + \frac{1}{2}mv_i^2,$$

$$v_f^2 = \frac{2\Delta KE}{m} + v_i^2,$$

$$v_f = \sqrt{\frac{2\Delta KE}{m} + v_i^2} = \sqrt{\frac{2 \cdot 383.15\,J}{12.0\,kg} + (2.50\,ms^{-1})^2} = 8.37\,ms^{-1}.$$

Answer:

1) The FBD is included into the answer.

2) $N = 110.5\,N$.

3) $F_{fr} = 33.15\,N$.

4) $W_g = -201.1\,J$.

5) $\Delta E_{int} = 165.75\,J$.

6) $W_{appl} = 750\,J$.

7) $\Delta KE = 383.15\,J$.

8) $v_f = 8.37\,ms^{-1}$.

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