

Answer on Question #60826 - Physics - Molecular Physics | Thermodynamics

Show that  $\left(\frac{\partial E}{\partial V}\right)_{\beta, N} + \beta \left(\frac{\partial p}{\partial \beta}\right)_{N, V} = -p$

Solution:

$$d\beta = -\frac{\beta}{T} dT \Rightarrow \beta \left(\frac{\partial p}{\partial \beta}\right) = -\frac{\beta T}{\beta} \left(\frac{\partial p}{\partial T}\right) = -T \left(\frac{\partial p}{\partial T}\right) = -T \left(\frac{\partial S}{\partial V}\right)$$

$$dE = TdS - pdV \Rightarrow \frac{\partial E}{\partial V} = T \left(\frac{\partial S}{\partial V}\right) - p \Rightarrow \frac{\partial E}{\partial V} - T \left(\frac{\partial S}{\partial V}\right) = \frac{\partial E}{\partial V} + \beta \left(\frac{\partial p}{\partial \beta}\right) = -p$$

$$\frac{\partial E}{\partial V} = 0 \text{ (Joule's law)}$$

$$\beta \left(\frac{\partial p}{\partial \beta}\right) = -T \left(\frac{\partial S}{\partial V}\right) = -T \left(\frac{\partial p}{\partial T}\right) = -T \frac{RdT}{VdT} = -R \frac{T}{V} = -p$$

$$0 - p = -p \Rightarrow \left(\frac{\partial E}{\partial V}\right) + \beta \left(\frac{\partial p}{\partial \beta}\right) = -p$$