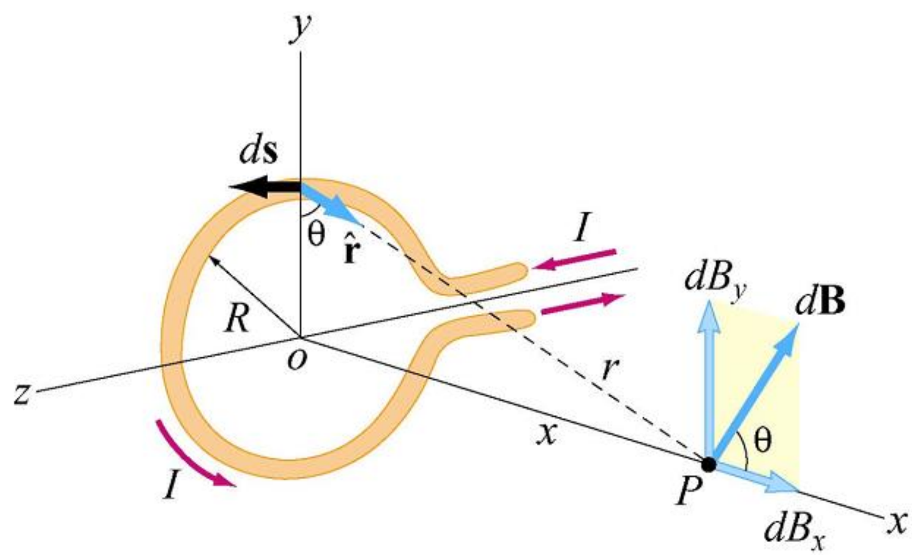
Answer on Question 60825, Physics, Electromagnetism

Question:

A 45 turn coil that has a radius of 0.183 meters has a current of $2.81\ A$. What is the strength of the magnetic field at a point on the coil's axis that is 0.483 meters from the center of the coil.

Solution:

Here's the sketch of our task:



We have a 45 turn coil (here in the sketch depicted only one loop of the coil for simplicity) that has a radius of $R = 0.183\ m$. The current that flows through that coil is equal to $I = 2.81\ A$. Our task is to find the strength of the magnetic field at the point P on the coil's axis that is $x = 0.483$ meters from the center of the coil.

We can find the strength of the magnetic field at a point P from the Biot-Savart's Law. It states that the magnetic field due to a current carrying element is directly proportional to the current and the vector product of length vector of the current element and the vector joining the current element and the point P where the magnetic field is to be found and inversely proportional to the square of the distance between the current element and the point P. Mathematically, it can be written as follows:

$$d\vec{\boldsymbol{B}} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\boldsymbol{s}} \times \hat{\boldsymbol{r}}}{r^2},$$

here, $\frac{\mu_0}{4\pi}$ is the proportionality constant, $\mu_0 = 4\pi \cdot 10^{-7}\ T \cdot \frac{m}{A}$ is the permeability of free space, I is the current, $d\vec{\boldsymbol{s}}$ is the length vector of the current element ds ($ds = d\vec{\boldsymbol{s}} \times \hat{\boldsymbol{r}}$), $\hat{\boldsymbol{r}}$ is the vector joining the current element and the point P.

We can find r from the Pythagorean theorem:

$$r = \sqrt{x^2 + R^2}.$$

The magnetic field dB has two components: $dB_y = dBsin\theta$ along the y-axis and $dB_x = dBcos\theta$ along the x-axis. During the integration around the loop the non-x components will be cancelled by the symmetry. Thus, we get only the x-component:

$$dB_x = dBcos\theta.$$

As we can see from the geometry of the task $cos\theta = \frac{R}{r}$.

Then, we get:

$$dB_x = dBcos\theta = \frac{\mu_0 I}{4\pi}\frac{d\vec{\boldsymbol{s}} \times \hat{\boldsymbol{r}}}{r^2} \cdot \frac{R}{r} = \frac{\mu_0 IR}{4\pi}\frac{ds}{r^3} = \frac{\mu_0 IR}{4\pi}\frac{ds}{(x^2 + R^2)^{\frac{3}{2}}}.$$

After integrating, we get:

$$B_x = \int_0^{2\pi R} dBcos\theta = \int_0^{2\pi R} \frac{\mu_0 IR}{4\pi}\frac{ds}{(x^2 + R^2)^{\frac{3}{2}}} =$$

$$= \frac{\mu_0 IR}{4\pi(x^2 + R^2)^{\frac{3}{2}}}\int_0^{2\pi R} ds = \frac{\mu_0 IR}{4\pi(x^2 + R^2)^{\frac{3}{2}}} \cdot 2\pi R = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{\frac{3}{2}}}.$$

Since we have N turns in the coil, we get:

$$B_x = \frac{\mu_0 NIR^2}{2(x^2 + R^2)^{\frac{3}{2}}} = \frac{4\pi \cdot 10^{-7}\ T \cdot \frac{m}{A} \cdot 45\ turns \cdot 2.81\ A \cdot (0.183\ m)^2}{2 \cdot ((0.483\ m)^2 + (0.183\ m)^2)^{\frac{3}{2}}} =$$
$$= 1.93 \cdot 10^{-5}\ T.$$

Answer:

$B_x = 1.93 \cdot 10^{-5}\ T.$