

Answer on Question #60593, Physics / Mechanics | Relativity

A satellite is placed in a circular orbit around Jupiter. The altitude of the satellite above the surface of Jupiter is 775km. Jupiter has a mass of 1.90×10^{27} kg and a radius of 7.14×10^7 m.

1. Determine the radius of motion of the satellite.
2. What force is providing the centripetal force necessary for the satellite to stay in orbit?
3. In what direction is the centripetal force always acting?
4. Using the equation for centripetal force, $F_c = m(v^2/R)$, and the gravitational force, $F_g = GmM/R^2$, derive the mathematical equation that allows you to calculate the orbital speed of the satellite.
5. Calculate the orbital speed of the satellite circling Jupiter using the equation derived in part 3. The value of $G = 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2$.

Find: $v - ?$

Given:

$$M = 1.9 \times 10^{27} \text{ kg}$$

$$r = 7.14 \times 10^7 \text{ m}$$

$$h = 0.0775 \times 10^7 \text{ m}$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$

Solution:

1. Determine the radius of motion of the satellite.

$$R = r + h \quad (1)$$

$$\text{Of (1)} \Rightarrow R = 7.2175 \times 10^7 \text{ m}$$

2. What force is providing the centripetal force necessary for the satellite to stay in orbit?

Gravitational interaction between Jupiter and satellite

3. In what direction is the centripetal force always acting?

To the center of orbit

4. Using the equation for centripetal force, $F_c = m(v^2/R)$, and the gravitational force, $F_g = GmM/R^2$, derive the mathematical equation that allows you to calculate the orbital speed of the satellite.

Newton's Second Law in scalar form:

$$F = ma \quad (2)$$

$$\text{Of (2)} \Rightarrow F_g = ma_c \quad (3),$$

where F_g – gravitational interaction between Jupiter and satellite,

m – mass of satellite,

a_c – centripetal acceleration

$$F_g = G \frac{mM}{(r+h)^2} \quad (4)$$

$$a_c = m \frac{v^2}{(r+h)} \quad (5)$$

$$(4) \text{ and } (5) \text{ in } (3): G \frac{mM}{(r+h)^2} = m \frac{v^2}{(r+h)} \quad (6)$$

$$\text{Of (6)} \Rightarrow v = \sqrt{\frac{GM}{(r+h)}} \quad (7)$$

error in condition: $F_c = m(v/R)$ wrong, $F_c = m(v^2/R)$ right

5. Calculate the orbital speed of the satellite circling Jupiter using the equation derived in part 3. The value of $G=6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.

$$\text{Of (7)} \Rightarrow v = 4,19 \times 10^4 \text{ m/s}$$

error in condition: derived in part 3 wrong, derived in part 4 right

Answer:

1. $R = 7,2175 \times 10^7 \text{ m}$

2. Gravitational interaction between Jupiter and satellite

3. To the center of orbit

4. $v = \sqrt{\frac{GM}{(r+h)}}$

5. $v = 4,19 \times 10^4 \text{ m/s}$