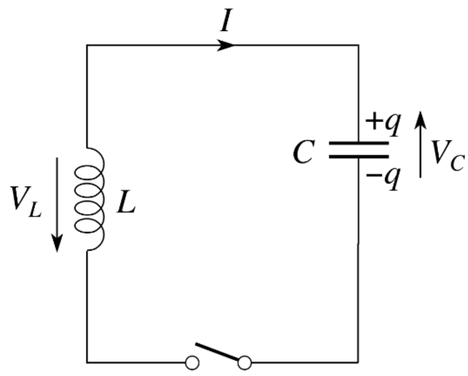
Answer on Question #60584, Physics – Electric Circuits

Obtain an expression for the frequency at which charge oscillates in an LC circuit.

Solution:



(a)

Let us consider some arbitrary time t after the switch is closed, so that the capacitor has a charge Q < Qmax and the current is I < Imax. At this time, both elements store energy, but the sum of the two energies must equal the total initial energy U stored in the fully charged capacitor at t=0:

$$U = U_C + U_L = \frac{q^2}{2C} + \frac{LI^2}{2}$$

Because we have assumed the circuit resistance to be zero, no energy is transformed to internal energy, and hence the total energy must remain constant in time.

This means that dU/dt = 0.

Therefore, by differentiating Equation 1 with respect to time while noting that q and I vary with time, we obtain

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{q^2}{2C} + \frac{LI^2}{2}\right) = \frac{q}{C}\frac{dq}{dt} + LI\frac{dI}{dt} = 0$$

We can reduce this to a differential equation in one variable by remembering that the current in the circuit is equal to the rate at which the charge on the capacitor changes: I = dq /dt . From this, it follows that $\frac{dI}{dt} = \frac{d^2q}{dt^2}$ .Substitution of these relationships into Equation 2 gives

$$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

The solution of this equation has the general form

$$q = q_{max}\cos(\omega t + \varphi)$$

where q_{max} is the maximum charge of the capacitor and the angular frequency ω is

To pop it back into the differential equation and see what happens

$$\frac{d^2}{dt^2}(q_{max}\cos(\omega t + \varphi)) = -\frac{1}{LC}q_{max}\cos(\omega t + \varphi)$$

$$-\omega^2 q_{max}\cos(\omega t+\varphi) = -\frac{1}{LC} q_{max}\cos(\omega t+\varphi)$$

Basically everything cancels but one parameter — angular frequency

$$\omega = \sqrt{\frac{1}{LC}}$$

An LC circuit is therefore an oscillating circuit. The frequency of such a circuit (as opposed to its angular frequency) is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$