Answer on Question #60279-Physics – Mechanics Relativity

A sonometer wire is stretched by a load of 6.48kg. Its vibrating portion has mass 0.5g. When vibrating in its fundamental mode, it gives 20 beats in 5 seconds with a tuning fork of frequency 256Hz. if the length of wire is slightly decreased, it is found to be in union with the fork. Calculate the original length of the wire.

Solution

$$n = \frac{1}{2l}\sqrt{\frac{Tl}{m}}$$

$$T = Mg$$

$$N = n_1 - n_2 = \frac{20}{5} = 4\ s^{-1}.$$

$$f = \frac{1}{2\pi}\sqrt{\frac{T(l-\Delta l)}{m}}$$

$$N = \frac{1}{2(l-\Delta l)}\sqrt{\frac{T(l-\Delta l)}{m}} - \frac{1}{2l}\sqrt{\frac{Tl}{m}} = \frac{1}{2}\sqrt{\frac{Mg}{m}}\left(\frac{1}{\sqrt{(l-\Delta l)}} - \frac{1}{\sqrt{(l)}}\right)$$

$$\frac{1}{\sqrt{(l-\Delta l)}} = \frac{1}{2\pi}\sqrt{\frac{T}{m}} = \frac{1}{2\pi f}\sqrt{\frac{Mg}{m}}$$

$$\frac{1}{\sqrt{(l)}} = \frac{1}{\sqrt{(l-\Delta l)}} - 2N\sqrt{\frac{m}{Mg}}$$

$$\frac{1}{\sqrt{(l)}} = \frac{1}{2\pi f}\sqrt{\frac{Mg}{m}} - 2N\sqrt{\frac{m}{Mg}} = \frac{1}{2\pi(256)}\sqrt{\frac{6.48(9.8)}{0.0005}} - 2(4)\sqrt{\frac{0.0005}{6.48(9.8)}} = 0.2215622 - 0.0224478$$
$$= 0.1991144$$

$$l = \frac{1}{0.1991144^2} = 25.2\ m.$$

Answer: $25.2\ m$.