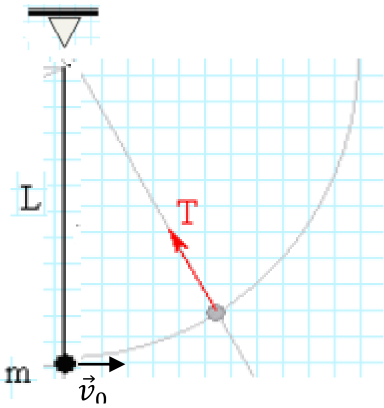
Answer on Question#60215 –Physics– Mechanics –Relativity

A heavy stone hanging from a massless string of length 15 m is projected horizontally with speed 147 m/s. The speed of the particle at the point where the tension in the string equals the weight of the particle is :

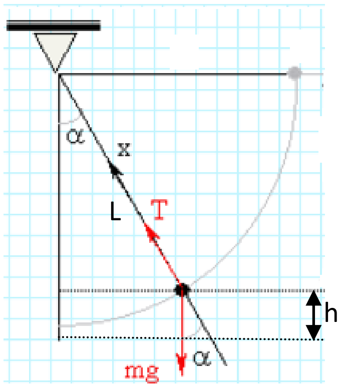
Solution. The stone moves under the action of gravity mg and the tension force of the thread T.



According to the statement of the problem $v_0 = 147$m/s, $L = 15$m. According to Newton's second law.

$$m\vec{a} = m\vec{g} + \vec{T}$$

We write it in the projection on the X-axis coinciding with the string at the moment when the string forms an angle α with the vertical (as shown in figure)



$$ma_x = -mgcos\alpha + T$$

As the stone moves in the radius of the circle L, a_x is the centripetal acceleration is equal to:

$$a_x = \frac{v^2}{L}$$

where v – velocity at this time. According to the statement of the problem $T = mg$. Hence

$$m\frac{v^2}{L} = -mgcos\alpha + mg \rightarrow \frac{v^2}{L} = g(1 - cos\alpha) \rightarrow 1 - cos\alpha = \frac{v^2}{gL}.$$

Using the law of conservation of energy:

In the initial moment of time the body will have only kinetic energy $\frac{mv_0^2}{2}$, at some point in time the body has both potential and kinetic energy $\frac{mv^2}{2} + mgh$, where $h = L - Lcos\alpha = L(1 - cos\alpha)$.)(solve right triangle shown in figure). Therefore law of conservation of energy

$$\frac{mv_0^2}{2} = \frac{mv^2}{2} + mgL(1 - cos\alpha).$$

$$\frac{v_0^2}{2} = \frac{v^2}{2} + gL(1 - cos\alpha) \rightarrow v_0^2 = v^2 + 2gL(1 - cos\alpha)$$

Using the formula $1 - cos\alpha = \frac{v^2}{gL}$ get $v_0^2 = v^2 + 2gL\frac{v^2}{gL} \rightarrow v_0^2 = 3v^2$.

$v = \frac{v_0}{\sqrt{3}} \approx 84.87$m/s.

Answer: $v = \frac{v_0}{\sqrt{3}} \approx 84.87$m/s.