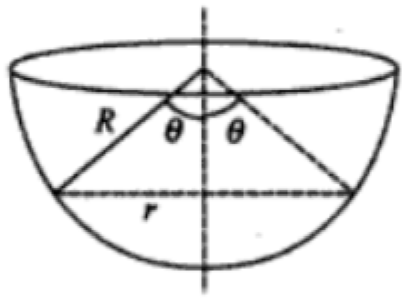
Answer on Question #60164, Physics Mechanics Relativity

A hemispherical bowl of radius R is rotated about its axis of symmetry which is kept vertical. A small block is kept in the bowl at a position where the radius makes an angle 0 with the vertical. The block rotates with the bowl without any slipping. The friction coefficient between the block and the bowl surface is u. Find the range of the angular speed for which the block will not slip.

Solution



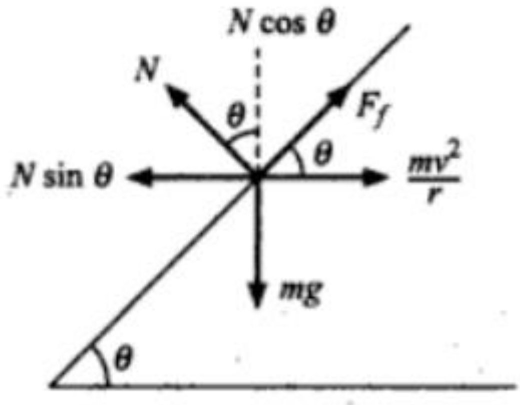
$$r = R\sin\theta.$$

Case (I). When the block tends up to slip down force of friction acts upward.

$$mg = N\cos\theta + F_f\sin\theta$$

$$mr\omega^2 = N\sin\theta - F_f\cos\theta$$

$$\frac{r\omega^2}{g} = \frac{N(\sin\theta - \mu\cos\theta)}{N(\cos\theta + \mu\sin\theta)} = \frac{\tan\theta - \mu}{1 + \mu\tan\theta}$$

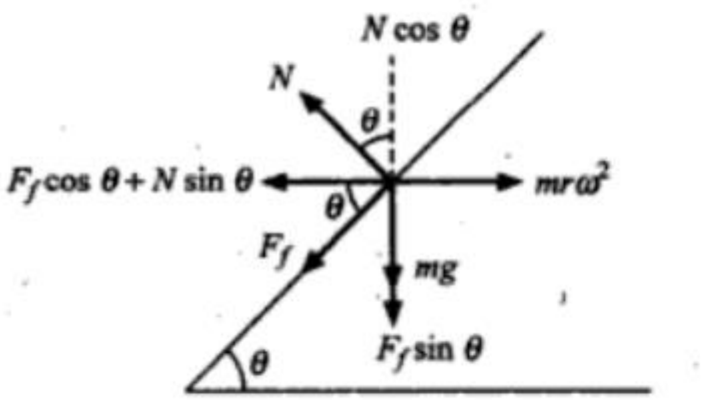


$$or\ \omega_{min} = \sqrt{\frac{g}{r}\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)} = \sqrt{\frac{g}{R\sin\theta}\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$$

Case (II). When the block tends up to slip upwards ($\omega \to \omega_{max}$), force of friction acts downwards. Therefore,

$$mg = N\cos\theta - F_f\sin\theta$$

$$mr\omega^2 = N\sin\theta + F_f\cos\theta$$



$$or\ \omega_{max} = \sqrt{\frac{g}{r}\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)} = \sqrt{\frac{g}{R\sin\theta}\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$$