## Answer on Question \#60162, Physics / Mechanics | Relativity |

18. A turn of radius 20 m is banked for the vehicles going at a speed of $36 \mathrm{~km} / \mathrm{h}$. If the coefficient of static friction between the road and the tyre is 0.4 , what are the possible speeds of a vehicle so that it neither slips down nor skids up?

## Solution:

Given:
$v=36 \frac{\mathrm{~km}}{\mathrm{hr}}=10 \mathrm{~m} / \mathrm{s}$,
$r=20 \mathrm{~m}$,
$\mu_{s}=0.4$
The road is banked with an angle,

$$
\theta=\tan ^{-1}\left(\frac{v^{2}}{r g}\right)=\tan ^{-1}\left(\frac{10^{2}}{20 * 10}\right)=26.57^{\circ}
$$

When the car travels at max speed it slips upward


Force equations at maximum speed v , at threshold of sliding up incline

$$
\begin{gathered}
\sum F_{x}=\frac{m v^{2}}{r}=N \sin \theta+\mu_{s} N \cos \theta \\
\sum F_{y}=0=N \cos \theta-\mu_{s} N \sin \theta-m g
\end{gathered}
$$

Solving the pair of equations for the maximum speed $v$ gives:

$$
\begin{gathered}
v_{\max }=\sqrt{\frac{r g\left(\sin \theta+\mu_{s} \cos \theta\right)}{\cos \theta-\mu_{s} \sin \theta}} \\
v_{\max }=\sqrt{\frac{20 \cdot 10 \cdot\left(\sin 26.57^{\circ}+0.4 \cdot \cos 26.57^{\circ}\right)}{\cos 26.57^{\circ}-0.4 \cdot \sin 26.57^{\circ}}}=15 \frac{\mathrm{~m}}{\mathrm{~s}}=15 \cdot 3.6 \frac{\mathrm{~km}}{\mathrm{hr}}=54 \frac{\mathrm{~km}}{\mathrm{hr}}
\end{gathered}
$$

For the case of sliding down

$$
\begin{aligned}
& \sum F_{x}=\frac{m v^{2}}{r}=N \sin \theta-\mu_{s} N \cos \theta \\
& \sum F_{y}=0=N \cos \theta+\mu_{s} N \sin \theta-m g
\end{aligned}
$$

Solving the pair of equations for the minimum speed v gives:

$$
\begin{gathered}
v_{\min }=\sqrt{\frac{r g\left(\sin \theta-\mu_{s} \cos \theta\right)}{\cos \theta+\mu_{s} \sin \theta}} \\
v_{\min }=\sqrt{\frac{20 \cdot 10 \cdot\left(\sin 26.57^{\circ}-0.4 \cdot \cos 26.57^{\circ}\right)}{\cos 26.57^{\circ}+0.4 \cdot \sin 26.57^{\circ}}}=4.085 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{gathered}=4.085 \cdot 3.6 \frac{\mathrm{~km}}{\mathrm{hr}}=14.7 \frac{\mathrm{~km}}{\mathrm{hr}} .
$$

Answer: $v_{\min }=14.7 \frac{\mathrm{~km}}{\mathrm{hr}}, v_{\max }=54 \frac{\mathrm{~km}}{\mathrm{hr}}$.

