Answer on Question #60013, Physics / Electromagnetism

a) Derive the expression for the group velocity of a uniform plane wave propagating in a good conductor and compare with phase velocity in the conductor

Solution:

Equation of plane wave:

$\vec{E} = \overrightarrow{E_{max}} \sin(\omega t - kx)$ (1),

where $\vec{E}$ – tension vector of electric field,

ω – angular speed,

t – time during which the wave propagates,

x – distance from the source to the observer,

k – wave number

Group velocity u describes the speed of the amplitude.

0f (1) $\Rightarrow$ write the condition of amplitude stability:

$t d\omega - x dk = const$ (2)

Find the derivative:

$d\omega dt - dk dx = 0$ (3)

Of (3) $\Rightarrow$ $u = \frac{dx}{dt} = \frac{d\omega}{dk}$ (4)

Group velocity u: $u = \frac{d\omega}{dk}$ (5)

The phase velocity v describes a monochromatic wave (unlimited sinusoid).

0f (1) $\Rightarrow$ write the condition of phase stability:

$\omega t - kx = const$ (6)

Find the derivative:

$\omega dt - k dx = 0$ (7)

Of (7) $\Rightarrow$ $v = \frac{dx}{dt} = \frac{\omega}{k}$ (8)

Phase velocity v:

$v = \frac{\omega}{k}$ (9)

Of (9) $\Rightarrow$ $\omega = vk$ (10)

(10) in (5): $u = \frac{d(vk)}{dk} = v + k\frac{dv}{dk}$ (11)

Wave number:

$k = \frac{2\pi}{\lambda}$ (12),

where λ – wavelength

Of (12) $\Rightarrow$ $dk = -\frac{2\pi}{\lambda^2} d\lambda$ (13)

Of (12) and (13) $\Rightarrow\ k \times \frac{dv}{dk} = \frac{2\pi}{\lambda} \times \left(-\frac{\lambda^2 dv}{2\pi d\lambda}\right) = -\lambda \frac{dv}{d\lambda}$ (14)

(14) in (11): $u = v - \lambda \frac{dv}{d\lambda}$ (15)

Rayleigh formula:

$u = v - \lambda \frac{dv}{d\lambda}$ (16)

Answer:

Group velocity: $u = \frac{d\omega}{dk}$

Phase velocity: $v = \frac{\omega}{k}$

Compare: $u = v - \lambda \frac{dv}{d\lambda}$