Answer on Question 59959, Physics, Electric Circuits

Question:

The electric field at a distance of $20\ cm$ from the centre of a uniformly charged dielectric sphere of radius $10\ cm$ is $100\ V/m$. Then, the electric field at $3\ cm$ distance from the centre of the sphere is:

a) $150\ V/m$

b) $125\ V/m$

c) $120\ V/m$

d) zero

Solution:

Let's find the formula for electric field inside and outside the sphere. Let's use the Gauss's Law. It states that the net electric flux through any closed surface is equal to $1/\varepsilon$ times the net electric charge within that closed surface:

$$\Phi_E = \frac{q}{\varepsilon_0},$$

here, Φ_E is the electric flux through a closed surface S enclosing any volume V, q is the total charge enclosed within S, and ε_0 is the permettivity of free space.

So, let's first find the electric field inside the sphere ($r < R$). In this case, the formula for the electric flux looks like:

$$\Phi_E(r) = \int E(r) \cdot dS = E(r) \int dS = 4\pi r^2 E(r),$$

here, $E(r)$ is the electric field, dS is the vector representing an infinitesimal element of area of the surface, symbol dot (·) represents the dot product of two vectors.

The charge of the sphere at the distance r from the centre of the sphere is equal to:

$$q(r) = V\rho = \frac{4}{3}\pi\rho r^3, (1)$$

here, V is the volume of the charged sphere, ρ is the volume charge density.

The charge of the sphere at the distance R from the centre of the sphere is equal to:

$$q(R) = Q = V\rho = \frac{4}{3}\pi\rho R^3, (2)$$

here, R is the radius of the sphere.

From the equations (1) – (2) we can find $q(r)$. Let's divide equation (1) by equation (2), we get:

$$q(r) = q(R)\frac{r^3}{R^3} = Q\frac{r^3}{R^3}.$$

Then, from the Gauss's law we get:

$$\Phi_E(r) = 4\pi r^2 E(r) = \frac{Qr^3}{\varepsilon_0 R^3}.$$

Solving for $E(r)$ we get the electric field inside the sphere ($r < R$):

$$E(r) = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

Similarly, we can find the electric field outside the sphere ($r > R$). In this case, the formula for the electric flux will be

$$\Phi_E(r) = 4\pi r^2 E(r) = \frac{Q}{\varepsilon_0}.$$

Solving for $E(r)$ we get the electric field outside the sphere ($r > R$):

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}.$$

Finally, we get:

$$E = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}, & r > R \\ \frac{Qr}{4\pi\varepsilon_0 R^3}, & r < R \end{cases},$$

Let's first find the charge of the sphere Q from the first equation:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2},$$

$$Q = 4\pi\varepsilon_0 E r^2 = 4\pi \cdot 8.85 \cdot 10^{-12}\ \frac{C}{V \cdot m} \cdot 100\ \frac{V}{m} \cdot (0.2\ m)^2 = 445 \cdot 10^{-12}\ C.$$

As we know the charge of the sphere Q, we can find the electric field at $r = 3\ cm$ distance from the centre of the sphere:

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3} = \frac{445 \cdot 10^{-12}\ C \cdot 0.03\ m}{4\pi \cdot 8.85 \cdot 10^{-12}\ \frac{C}{V \cdot m} \cdot (0.1\ m)^3} = 120\ \frac{V}{m}.$$

Answer:

c) $120\ V/m$